Automatic Estimation of Initial Approximations of Parameters for Bundle Adjustment

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Abstract

The bundle adjustment has been widely used in calibration and orientation camera. However, because the observation equations are non-linear, approximations of parameters are necessary at the beginning of the adjustment. This paper describes a method to calculate the approximations of exterior orientation parameters of photographs and coordinates of object points associated with any model and/or any object space coordinate system. This method is based on relative orientation of a stero pair by the linear coplanarity condition and the unique decomposition of rotation matrices to angular elements. Established independent models are then connected to a global model. If necessary, the global model coordinate system is transformed to any object coordinate system automatically through singular value decomposition of rotation matrices. This method realizes semi-automatic bundle adjustment and camera calibration without control points. The discussion is validated by two detailed experiments.

Introduction

The bundle adjustment has been widely used in camera calibration and aerotriangulation. However, because the observation equations are non-linear, approximations of all parameters are required at the beginning of computation.

In close-range photogrammetry, the approximations of exterior orientation parameters are usually recorded at exposure stations, and object-point coordinates are measured by other means. These procedures are time-consuming and sometimes inconvenient, because a convergent or parallel imaging configuration rather than vertical one is often used. For a digital plotter (digital-image-based plotter) which is now being developed in many organizations (Lohmann, 1989; Ohtani, 1989; Miller, 1992), easy manipulation is essential for operators who are not familiar with photogrammetry. Hence, an automatic or semi-automatic adjustment procedure is now strongly called for.

To this end, linear solutions based on the direct linear transformation (DLT) method (Abdel-Aziz, 1971; Naftel, 1991) are often in use, which relate comparator coordinates to ground control points (GCP) directly. On the other hand, a closed form of space resection was derived by Zeng (1992). Both solutions are mainly intended for orientation of single photograph and require a minimum of six and three three-dimensional (3D) control points, respectively. On the other hand, Tsai (1986) discussed an automatic orientation of a stereo pair with two steps. His method requires 2D control points. These methods are not practical for the orientation of multiple overlapping photographs. For a large object-space, many control points have to be set out to cover it with multiple photographs.

We have recently developed a method to calculate automatically the approximations of exterior orientation parameters and coordinates of object points associated with any model coordinate system from pass-points of photographs. If necessary, the model coordinate system is automatically transformed to any operator-assigned object space coordinate system. The method is based on relative orientation of each stero pair using the linear coplanarity condition and subsequent connection of models (Hattori, 1992).

The authors' method solves the following problems:

- Automated orientation. This is very useful for digital plotters, because users do not have to learn deeply the orientation theory and can easily define an object coordinate system on the screen, while observing the model stereoscopically.
- Industrial measurement with only scale or level controls.
- Camera calibration without control points.

It has been shown that cameras can be calibrated with only the coplanarity condition (Fraser, 1982).

Outline of Evaluation of Initial Values of Parameters

Figure 1 shows an example of an imaging configuration in camera calibration which will be referred to in the experiments again. Three-dimensionally allocated targets are imaged convergently at various positions with various camera rotations. Approximations of parameters are estimated with the following processes:

- Overlapping photographs are each separated into an independent (local) model. Rotation matrices of independent models are evaluated and decomposed to angular elements.
- The independent models are linked to form a global model.
- If necessary, the global model coordinate system is transformed to an object space coordinate system using some control (types of control can be different according to objects and purposes).
- Model or object space coordinates of target points are calculated. Finally, the rotation matrix of each photograph in the model or object space coordinate system is decomposed to angular elements.

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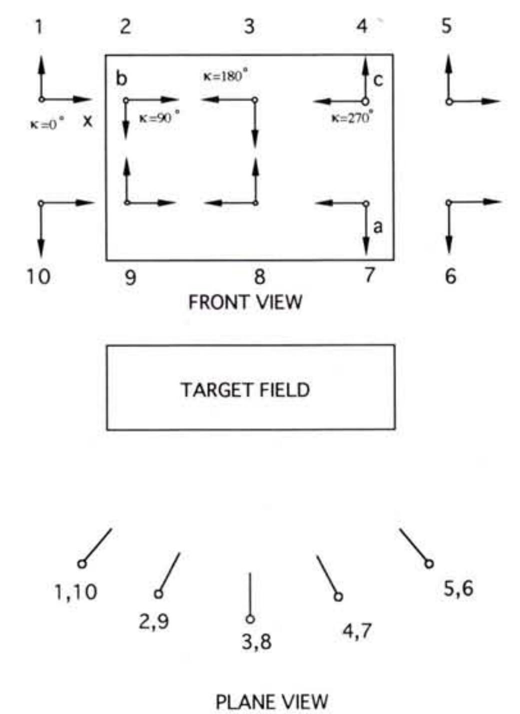

Figure 1. Imaging configuration for camera calibration.

Relative Orientation by the Linear Coplanarity Condition

Coplanarity Condition

First, we start with a pair of overlapping photographs. The interior orientation is assumed complete. Model coordinates of two cooresponding points in Figure 2-1 are expressed as

$$\begin{bmatrix} X_{p1} \\ Y_{p1} \\ Z_{p1} \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ y_1 \\ -c \end{bmatrix}$$

$$\begin{bmatrix} X_{p2} \\ Y_{p2} \\ Z_{p2} \end{bmatrix} = \begin{bmatrix} n_{11} & n_{12} & n_{13} \\ n_{21} & n_{22} & n_{23} \\ n_{31} & n_{32} & n_{33} \end{bmatrix} \cdot \begin{bmatrix} x_2 \\ y_2 \\ -c \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

where $(x_1\ y_1\ -c)^T$, $(x_2\ y_2\ -c)^T$ are photographic coordinates, $(X_{p1}\ Y_{p1}\ Z_{p1})^T$, $(X_{p2}\ Y_{p2}\ Z_{p2})^T$ are model coordinates, c is camera focal length, and B is the base length (unity with unknown sign).

The following coplanarity condition

$$Yp_1Zp_2 - Zp_1Yp_2 = 0 \tag{2}$$

of the model coordinates can also be written as

$$\begin{aligned} &p_1x_1x_2+p_2x_1y_2+p_3x_1(-c)+q_1y_1x_2+q_2y_1y_2+q_3y_1(-c) \\ &+r_1(-c)x_2+r_2(-c)y_2+r_3(-c)(-c) = 0 \end{aligned} \tag{3}$$

where

$$\begin{aligned} &p_1 = m_{21}n_{31}-m_{31}n_{21},\ p_2 = m_{21}n_{32}-m_{31}n_{22},\ p_3 = m_{21}n_{33}-m_{31}n_{23}, \\ &q_1 = m_{22}n_{31}-m_{32}n_{21},\ q_2 = m_{22}n_{32}-m_{32}n_{22},\ q_3 = m_{22}n_{33}-m_{32}n_{23}, \\ &r_1 = m_{23}n_{31}-m_{33}n_{21},\ r_2 = m_{23}n_{32}-m_{33}n_{22},\ r_3 = m_{23}n_{33}-m_{33}n_{23}, \end{aligned} \tag{4}$$

It can be easily seen that a vector

$$\mathbf{a} = (p_1p_2p_3q_1q_2q_3r_1r_2r_3)^T$$

has a relation

$$\mathbf{a}^T\mathbf{a} = 2.$$

Expressing Equation 3 in the form of an observation equation

$$\mathbf{Xa} = \mathbf{v}, \tag{5}$$

where $\mathbf{X}$ is a design matrix and $\mathbf{v}$ is a residual vector, one can solve $\mathbf{a}$ by minimizing $\mathbf{v}^T\mathbf{v}$. After introducing a Langrangian multiplier u for constraint minimization, the objective function becomes

$$U = \mathbf{a}^T\mathbf{X}^T\mathbf{Xa} - u(\mathbf{a}^T\mathbf{a} - 2). \tag{6}$$

By differentiating Equation 6 with a, one gets

$$(\mathbf{X}^T\mathbf{X}-u\mathbf{I})\mathbf{a} = 0 \tag{7}$$

where $\mathbf{a}$ is an eigen-vector and u is a variance of residuals, i.e., $\|v\|^2/2$. Even if the imaging configuration is good, one or more of u's that are near zero may be obtained. The authors still can not answer how many candidates of u should be obtained in terms of the specific imaging quality. On the other hand, the threshold for u can be estimated from the threshold for y-parallaxes as follows:

From Equations 1, y-parallax (Dy) of a model point in the image space can be written as

$Dy=(Yp_1{\cdot}Zp_2 - Yp_2{\cdot}Zp_1){\cdot}c/\ (Zp_1{\cdot}Zp_2)$

which can be approximated by

$Dy \approx (Yp_1{\cdot}Zp_2 - Yp_2{\cdot}Zp_1)/c$

Threshold by Dy can be defined as

$\mathrm{Th}_{Dy} \geq \sqrt{((1/n){\cdot}\Sigma Dy^2)}=\sqrt{((1/n{\cdot}c^2){\cdot}\Sigma(Yp_1{\cdot}Zp_2 - Yp_2{\cdot}Zp_1)^2)}$,

where Th_{Dy} is the threshold for y-parallaxes, n is number of image points of the model. Recalling that u is the variance of residuals $\|v\|^2/2$, one can write

$\mathrm{Th}_u \geq (1/2){\cdot}(\mathrm{Th}_{Dy})^2{\cdot}(n){\cdot}(c^2)$

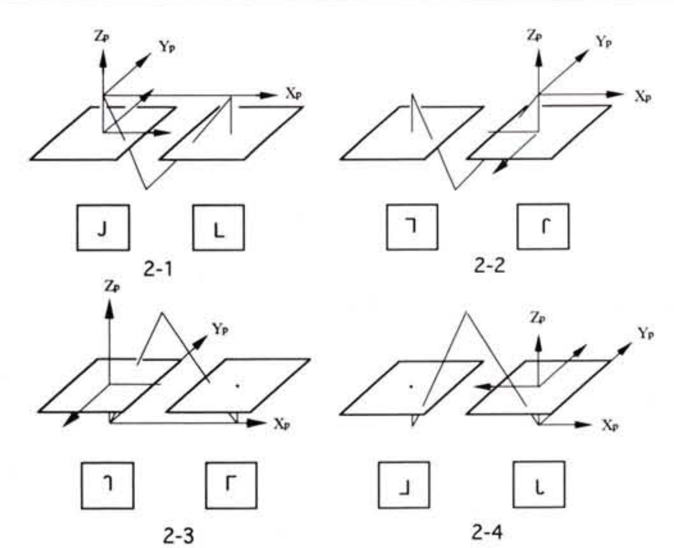

Figure 2. Four solutions retrieved from the coplanarity condition.

where Th_u is the threshold for u. Out of multiple candidates of u obtained after screening with the threshold, the correct one is determined by the following procedure.

Determination of the Rotation Matrices and Angles

The rotation matrices $(\mathbf{m}_{ij})$ and $(\mathbf{n}_{ij})$ are evaluated from the vector **a**. Even though Figure 2-1 is assumed to be correct, Figures 2-2, 2-3, and 2-4 as well as Figure 2-1 are included in solutions. Figures 2-1 and 2-2 are equivalent, whereas Figures 2-3 and 2-4 are false, because they are turned over into a negative position. The rotation matrices must be defined as

$$\mathbf{m}_{ij} = \begin{bmatrix} \cos\varphi_1 & 0 & -\sin\varphi_1 \\ 0 & 1 & 0 \\ \sin\varphi_1 & 0 & \cos\varphi_1 \end{bmatrix} \cdot \begin{bmatrix} \cos\kappa_1 & \sin\kappa_1 & 0 \\ -\sin\kappa_1 & \cos\kappa_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{8-1}$$

$$= \begin{bmatrix} \cos\varphi_1\ \cos\kappa_1 & \cos\varphi_1\ \sin\kappa_1 & -\sin\varphi_1 \\ -\sin\kappa_1 & \cos\kappa_1 & 0 \\ \sin\varphi_1\ \cos\kappa_1 & \sin\varphi_1\ \sin\kappa_1 & \cos\varphi_1 \end{bmatrix}$$

$$(\mathbf{n}_{ij}) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\omega_2 & \sin\omega_2 \\ 0 & -\sin\omega_2 & \cos\omega_2 \end{bmatrix} \begin{bmatrix} \cos\varphi_2 & 0 & -\sin\varphi_2 \\ 0 & 1 & 0 \\ \sin\varphi_2 & 0 & \cos\varphi_2 \end{bmatrix} \begin{bmatrix} \cos\kappa_2 & \sin\kappa_2 & 0 \\ -\sin\kappa_2 & \cos\kappa_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\varphi_2\ \cos\kappa_2 & \cos\varphi_2\ \sin\kappa_2 & -\sin\varphi_2 \\ -\cos\omega_2\ \sin\kappa_2 + \sin\omega_2\ \sin\varphi_2\ \cos\kappa_2 & \cos\omega_2\ \cos\kappa_2 + \sin\omega_2\ \sin\varphi_2\ \sin\kappa_2 & \sin\omega_2\ \cos\varphi_2 \\ \sin\omega_2\ \sin\kappa_2 + \cos\omega_2\ \sin\varphi l_2\ \cos\kappa_2 & -\sin\omega_2\ \cos\kappa_2 + \cos\omega_2\ \sin\varphi_2\ \sin\kappa_2 & \cos\omega_2\ \cos\varphi_2 \end{bmatrix} \tag{8-2}$$

It should be noted that the rotation order in the definition is unique. For other orders, it can be shown that there are some angles at which the rotation matrix becomes singular and fails to be decomposed to angular elements.

Evaluation of ϕ_1

Because $m_{23} = 0$, from Equations 4

$$m_{33}n_{21} = -r_1,\ m_{33}n = -r_2,\ m_{33}n_{23} = -r_3 \tag{9}$$

And then

$m_{33}{}^2(n_{21}{}^2+n_{22}{}^2+n_{23}{}^2) = r_1{}^2+r_2{}^2+r_3{}^2$

Because the photographs are assumed diapositives, $m_{33} > 0$. From the orthogonality of $\mathbf{n}_{ij}$,

$$m_{33} = \sqrt{(r_1^2 + r_2^2 + r_3^2)} \tag{10}$$

From Equation 10 two candidates of ϕ_1 are obtained. The correct one is not evaluated at this stage. Then, from Equations 9

$$n_{21} = -r_1/m_{33},\ n_{22} = -r_2/m_{33},\ n_{23} = -r_3/m_{33} \tag{11}$$

Multiplying the first, second, and third of Equations 4 with n_{21}, n_{22}, and n_{23}, respectively, and summing them up, one obtains

$$m_{31} = -(p_1n_{21} + p_2n_{22} + p_3n_{23}). \tag{12-1}$$

Likewise, one gets

$$m_{32} = -(q_1n_{21}+q_2n_{22}+q_3n_{23}) \tag{12-2}$$

$$m_{33} = -(r_1n_{21}+r_2n_{22}+q_3r_{23}) \tag{12-3}$$

where Equation 12-3 is identical to Equation 10.

Evaluation of κ_1

Writing the first six expressions of Equations 4 in the form of $m_{21}n_{31}=p_1+m_{31}n_{21}$, $m_{21}n_{32}=p_2+m_{31}n_{22}$, $m_{21}n_{33}=p_3+m_{31}n_{23}$, $m_{22}n_{31}=q_1+m_{32}n_{21}$, $m_{22}n_{32}=q_2+m_{32}n_{22}$, $m_{22}n_{33}=q_3+n_{32}n_{23}$, after multiplying the first with the fourth, the second with the fifth, and the third with the sixth of each side of the above expressions and summing them, one can calculate the right side of it. And the left side becomes

$m_{21}m_{22}(n_{31}{}^2+n_{32}{}^2+n_{33}{}^2)=m_{21}m_{22}=-\sin\kappa_1\cos\kappa_1= -(1/2)\sin2\kappa_1$

This procedure produces four candidates for k_1.

Then n_{31}, n_{32}, and n_{33} are evaluated for each candidate of k_1 using the following different equations for better precision:

(a) for $-3/4\ \pi \le \kappa_1 < -\pi/4$ or $\pi/4 \le \kappa_1 < 3/4\ \pi$

$$n_{31} = (p_1+m_{31}n_{21})/(-\sin\kappa_1),\ n_{32} = (p_2+m_{31}n_{22})/(-\sin\kappa_1), \quad n_{33} = (p_3+m_{31}n_{23})/\sin\kappa_1 \tag{13-1}$$

(b) for $-\pi/4 \le \kappa_1 < \pi/4$ or $3/4\ \pi \le \kappa_1 \le 5/4\ \pi$

$$n_{31} = (q_1+m_{32}n_{21})/\cos\kappa_1),\ n_{32} = (q_2+m_{32}n_{22})/\cos\kappa_1, \quad n_{33} = (q_3+m_{32}n_{23})/\cos\kappa_1 \tag{13-2}$$

Evaluation of φ_2, ω_2

From Equation 8-2

$$\sin\omega_2\cos\varphi = n_{23},\ \cos\omega_2\ \cos\varphi_2 = n_{33}. \tag{14}$$

Because $n_{33} > 0$, which means $\cos\varphi_2 \neq 0$,

$$\cos\varphi_2 = \sqrt{(n_{23}^2 + n_{33}^2} \tag{15}$$

There exist four candidates for φ_2. And for each candidate of φ_2, angle ω_2 is evaluated by

$$\sin\omega_2 = n_{23}/\cos\varphi_2,\ \cos\omega_2 = n_{33}/\cos\varphi_2. \tag{16}$$

Evaluation of κ_2

Rewriting Equation 8-2 in the form of

$$\begin{aligned} (-\cos\omega_2)\sin\kappa_2 + (\sin\omega_2\ \sin\varphi_2)\cos\kappa_2 &= n_{21} \\ (\cos\omega_2)\cos\kappa_2 + (\sin\omega_2\ \sin\varphi_2)\sin\kappa_2 &= n_{22} \\ (\sin\omega_2)\sin\kappa_2 + (\cos\omega_2\ \sin\varphi_2)\cos\kappa_2 &= n_{31} \\ (-\sin\omega_2)\cos\kappa_2 + (\cos\omega_2\ \sin\varphi_2)\sin\kappa_2 &= n_{32}, \end{aligned} \tag{17}$$

one solves the first two equations to get $\sin\kappa_2$ and $\cos\kappa_2$. They are always solvable, even if $\sin\varphi_2$ is zero. And this κ_2 is tested by substituting it into the third and fourth equations. Those sets of candidates of φ_2 and ω_2 which fail to satisfy both equations are abandoned.

Strict Relative Orientation and Determination of the Sign of the Base Length

Because the precision of approximations evaluated above is usually not sufficient, one should execute relative orientation again using these approximations. An independent model is thus obtained, which is either Figure 2-1 or 2-2.

Next the sign of the base length is determined in such a way that, if *Zp* coordinates of objects in the independent model coordinate system are negative, it is set plus, and if *Zp* coordinates are positive, i.e., reverse from actual configuration, it is set minus.

Evaluation of Orientation Parameters in the Object Space Coordinate System

Model Connection in the Global Model Coordinate System

Independent (local) models thus formed are linked to make a global model by usual successive orientations. To this end, one independent model is selected as a datum model (a reference of the global model). Scales of successive models are adjusted by scaling the respective base lengths. As a result, exposure positions and the rotation matrices associated with the global coordinate system $X_M\ Y_M\ Z_M$ are determined.

Transformation from the Global Model Coordinate System to the Object Space Coordinate System

When an object space coordinate system ($X\ Y\ Z$) is given, global model coordinates $X_M Y_M Z_M$ are further transformed to the object coordinates. If no object space coordinate system is given in 3D plotting, the operator has to set it arbitrarily. With a digital plotter, one could easily define it on the screen with the help of an interactive CAD display.

Further consideration is given to the case in which the object space coordinate system is implicitly defined in the form of three or more 3D control points. In most industrial applications, this will be the common case. In aerial photogrammetry, however, planimetric controls and height controls are usually separated. In such cases, the usual affine transformation would deliver a sufficient solution.

The closed-form problem of absolute orientation has already been solved by some researchers (Schut, 1960; Arun, 1987; Horn, 1988). Schut expressed the rotation matrix by quarternions and derived rotational elements using three 3D control points. The methods of Arun and Horn are equivalent. Using three or more 3D control points, both researchers derived the optimal rotation matrix by least squares utilizing the orthonormal property of the matrix. Arun employed the singular-value decomposition, while Horn used the eigenvalue-eigenvector decomposition with quarternions. The method by Arun is adopted in our approach as follows. The similarity transformation,

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = S \cdot \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \cdot \begin{bmatrix} X_M \\ Y_M \\ Z_M \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix}, \qquad (18)$$

is commonly used for 3D space transformation, where S is a scale, $\mathbf{A} = (\mathbf{A}_{ij})$ is an orthogonal matrix and $\mathbf{B} = (\mathbf{B}_i)$ is a translation vector. $\mathbf{B}$ and S are evaluated from the coordinates of gravity centers and the scale ratio of two coordinate systems. Thus, Equation 18 is reduced to the form

$$\mathbf{X}_i = \mathbf{A}\,\mathbf{X}_{Mi},\ (\text{for } i = 1,2,\ldots n) \qquad (19)$$

where suffix i means control point number. $\mathbf{X}_i$ and $\mathbf{X}_{Mi}$ are coordinate vectors associated with the object space coordinate system and the global model coordinate system, respectively. It is assumed that their origins are already shifted to respective gravity centers and $\mathbf{X}_i$ are being scaled by S. The matrix $\mathbf{A}$ is determined so as to minimize

$$F = \sum_{i=1}^{n} (\mathbf{A}\,\mathbf{X}_{Mi} - \mathbf{X}_i)^T (\mathbf{A}\,\mathbf{X}_{Mi} - \mathbf{X}_i) \qquad (20)$$

After expanding and rearranging, it follows that

$$F = \sum_{i=1}^{n} (\mathbf{X}_i^T\mathbf{X}_i + \mathbf{X}_{Mi}^T\,\mathbf{X}_{Mi} - 2\mathbf{X}_{Mi}^T\,\mathbf{A}^T\,\mathbf{X}_i) \qquad (21)$$

is minimized when

$$Trace(\sum_{i=1}^{n} (\mathbf{X}_i\,\mathbf{A}^T\,\mathbf{X}_{Mi}^T)) = Trace\,(\mathbf{A}^T \sum_{i=1}^{n} (\mathbf{X}_i\mathbf{X}_{Mi}^T))$$

is maximized. With appropriate orthogonal matrices $\mathbf{U}$, $\mathbf{V}$ which singular-value-decompose $N = \Sigma\,(\mathbf{X}_i\mathbf{X}_{Mi}{}^T)$ in such a way that

$$N = \sum_{i=1}^{n} (\mathbf{X}_i\mathbf{X}_{Mi}^T) = \mathbf{V}\cap\mathbf{U}^T, \qquad (22)$$

where $\cap$ is a diagonal matrix with positive elements. The optimal solutionof the matrix $\mathbf{A}$ is given as

$$\mathbf{A} = \mathbf{V}\,\mathbf{U}^T. \qquad (23)$$

The orthogonal matrices $\mathbf{U}$ and $\mathbf{V}$ can be determined in the following way. Let $\mathbf{E}$ be a diagonal matrix with elements of eigenvalues of matrix $\mathbf{N}^T\mathbf{N}$, and $\mathbf{U}$ be a matrix with corresponding eigenvectors as column-vectors. Thus,

$$\mathbf{N}^T\mathbf{N} = \mathbf{U}\,\mathbf{E}\,\mathbf{U}^T.$$

Then the matrix $\mathbf{V}$ is defined as

$$\mathbf{V} = \mathbf{N}\,\mathbf{U}\,\mathbf{E}^{-1/2},$$

where $\mathbf{E}^{-1/2}$ is a diagonal matrix, the elements of which are inverse square root of respective elements of $\mathbf{E}$. Because the eigenvalues of $\mathbf{N}^T\mathbf{N}$ must be positive, this is always executable. If any eigenvalue is zero, it means that the object space is degenerated, i.e., there is a bad distribution of control points. Then $\cap$ in Equation 22 and $\mathbf{A}$ in Equation 23 become respectively

$$\cap = \mathbf{V}^T\,\mathbf{N}\,\mathbf{U} = \mathbf{E}^{1/2} \text{ and}$$
$$\mathbf{A} = \mathbf{N}\,\mathbf{U}\,\mathbf{E}^{-1/2}\,\mathbf{U}^T$$

Evaluation of Angular Elements

After all rotation matrices ($\mathbf{M}_{ij}$) associated with the object space coordinate system (or global model coordinate system) are determined, they are decomposed further to obtain angular elements. Let the matrices related to angular elements K, Φ, and Ω be simply denoted by $[\mathbf{K}]$, $[\Phi]$, and $[\Omega]$. In close-range photogrammetry, objects are imaged from various directions. If the rotation order of angles is fixed, the matrix ($\mathbf{M}_{ij}$) can be singular for some angles and unable to be decomposed to unique angular elements. In order to assure unique decomposition, one has to change the order of rotations depending on the values of elements of the rotation matrix. A typical definition is

(a) If $M_{13} \not\approx \pm 1$, $(\mathbf{M}_{ij}) = [\Omega][\Phi][\mathbf{K}]$

(b) If $M_{13} \approx \pm 1$ and $\mathbf{M}_{31} \not\approx \pm 1$, $(\mathbf{M}_{ij}) = [K][\Phi][\Omega]$

(c) If $M_{13} \approx \pm 1$ and $\mathbf{M}_{31} \approx \pm 1$, $(\mathbf{M}_{ij}) = [K][\Omega][\Phi]$

where the threshold for ($\not\approx \pm 1$) may be 0.8.

Because the treatments for all cases are similar, the discussion will be given only to case (a). Because $\sin\Phi = -M_{13}$, one gets two candidates of Φ for $-\pi < \Phi \leq \pi$. Because $\cos\Phi \neq 0$,

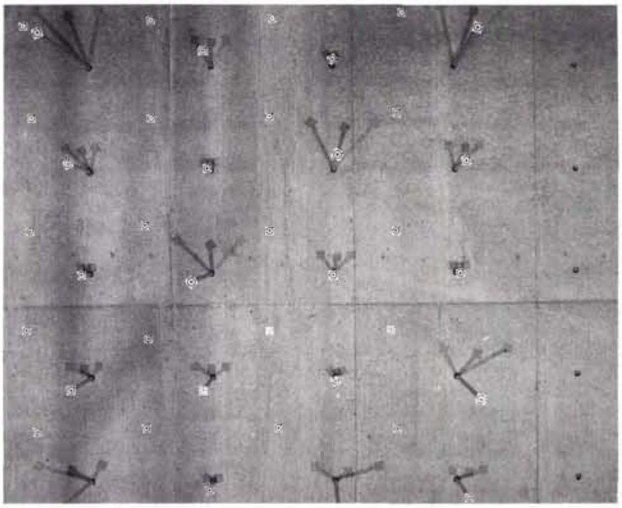

Figure 3. Left photograph used in the relative orientation.

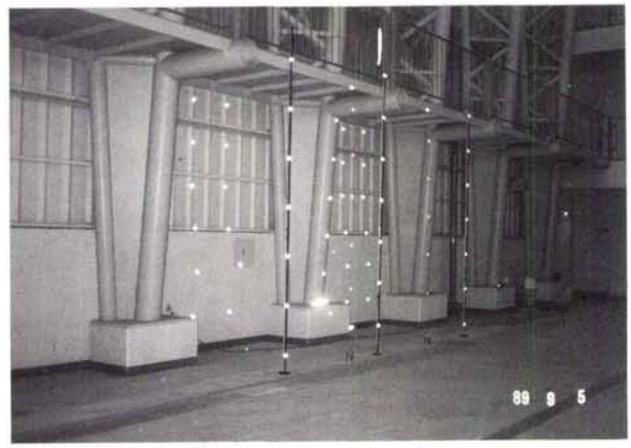

Figure 4. Target field for the camera calibration.

TABLE 1. APPROXIMATIONS AND THE MOST PROBABLE VALUES (MPV) OF RELATIVE ORIENTATION PARAMETERS

angles (deg·dec)	approx.	MPV
φ_1	349.00	358.96
κ_1	345.00	360.33
ω_2	−2.43	0.04
φ_2	0.00	−1.12
κ_2	−1.49	0.13

$$\begin{aligned}\sin \Omega &= M_{23}/\cos \Phi,\ \cos \Omega = M_{33}/\cos\Phi,\\ \cos K &= M_{11}/\cos \Phi,\ \sin K = M_{12}/\cos\Phi.\end{aligned} \tag{24}$$

For each candidate of Φ, Ω and K are determined uniquely. They are then tested as to whether they satisfy the following equations:

$$\begin{aligned}-\cos \Omega \sin K + \sin \Omega \sin \Phi \cos K &= M_{21}\\ \cos \Omega \cos K + \sin \Omega \sin \Phi \sin K &= M_{22}\\ \sin \Omega \sin K + \cos \Omega \sin \Phi \cos K &= M_{31}\\ -\sin \Omega \cos K + \cos \Omega \sin \Phi \sin K &= M_{32}\end{aligned} \tag{25}$$

Sets of candidates which do not satisfy all these equations within a certain tolerance (0.3 in our experiments) are discarded.

Experiments

The proposed procedure was applied in two experiments for a validity check, i.e., a simple relative orientation of a pair of stereo photographs and a camera calibration without control points.

Relative Orientation of a Pair of Stereo Photographs

A target field of 5 m by 5 m by 0.5 m (depth) was imaged by a 35-mm metric camera, Pentax Pams 645, c = 44.979mm. Two photographs were taken vertically in stereo with a base length of 1.5m, overlapping each other by 50 percent. Common pass-points are 12 in number (the minimum requirement is eight) which are distributed unbiasedly. Figure 3 is a left side photograph of the stero pair. This configuration is not ideal for automatic adjustment but very usual in industrial photogrammetry. Lens distortions were corrected using the parameters offered by the camera manufacturer.

Out of nine eigen-values obtained from Equation 7, three of them were 0.0598, 0.146, and 1.02, while the others are greater than 100,000. As a result of applying the procedure mentioned above to three small eigenvalues, a set of rotation angles with respect to the model coordinate system were obtained only for the third minimum. And the others did no produce false solutions. Residuals RMS y-parallaxes of 7 µm were obtained in the ensuing precise orientation. Table 1 shows the approximations and the most probable values (MPV) of the angles.

Camera Calibration without Control Points

The target field shown in Figure 4, was imaged by a metric camera, Geodetic Service CRC1, with $c \approx$ 240.0 mm (variable) and a film size of 23 cm by 23 cm. The camera is designed to determine precise coordiantes of object points by simultaneous adjustment with all other parameters, i.e., interior orientation parameters of the camera and exterior orientation parameters of the photographs (Fraser, 1982).

The target field was 4 m (height) by 5 m (width) by 2 m (depth) in size. 63 Sixty-three points were allocated three dimensionally over the area. Most of the points were imaged in most of the photographs.

As shown in Figure 1, ten photographs were taken with a kappa rotation of 90 degrees to each other. The distance from exposure stations to the field center is about 6 m. The linkage order of photographs adopted in the experiment is shown in Figure 5. Photographs 3 and 8 make a datum model (a reference of the global model coordinate system), and others are linked to this model. Pairs of the photographs (3,5), (3,4), (3,2), (3,1), (8,6), (8,7), (8,9), and (8,10) formed independent models. Models from (8,6) through (8,10) are linked to the datum model successively. According to additional experiments, however, any other combination of photographs also results in stable models as long as their convergent angles were not close to 90 degrees. The base length of the datum moel was set to unity (1 m).

```
3--8--6
|  |--7
|  |--9
|  |--10
|
|--5
|--4
|--2
|--1
```

Figure 5. Photo. connection, photo-pair 3-8 as datum model.

The model of interior orientation used is very general one, i.e., corrections Dx and Dy to measured plate coordinates x and y are given as

$$Dx = (K_1r^2+K_2r^4)(x-x_0)+P_1(r^2+2(x-x_0)^2)+2P_2(x-x_0)(y-y_0) \quad (26)$$
$$Dy = (K_1r^2+K_2r^4)(y-y_0)+2P_1(x-x_0)(y-y_0)+P_2(r^2+2(y-y_0)^2)$$

where x_0, y_0 are principal point coordinates, K_1, K_2 and P_1, P_2 are coefficients of radial and tangential lens distortions, respectively, and

$$r^2 = (x-x_0)^2+(y-y_0)^2.$$

All the approximations of interior orientation parameters except for a camera focal length were set to zeros. That of the camera focal length was read out by a micrometer-based indicator of the camera.

From Equation 7, one to two valid eigenvalues were obtained for all independent models. With threshold value (Th_u) of 1.7×10^7, these values ranged from 30 to 1.5×10^7. However, relative orientation parameters of all models were uniquely determined from the minimum eigenvalues. False solutions did not appear.

The calibration was executed in the global model coordinate system. In the case of no control points, either the free-network or the minimal constraint method can be employed to cope with seven rank deficiency. The authors adopted the latter. Seven degrees of freedom was fixed by giving infinite precision to Z_M of point a and to X_M, Y_M, Z_M of points b and c in Figure 1. Consequently, Tables 2 and 3 were obtained, which include the approximations and the adjusted values of the interior orientation parameters of the camera and the exterior orientation parameters for photo 1 and 10 as well as the RMS difference between approximations and adjusted values of target point coordinates. The exterior orientation parameters and the RMS value are given in global model coordinates. The model scale is about a half of the actual space.

Tables 1, 2, and 3 prove that the algorithm produces approximations of parameters suitable enough for further bundle adjustment.

Conclusion

This paper has discussed the algorithm for automatic calculation of approximations of parameters for bundle adjustment. Relative orientation parameters of each pair of photographs are estimated from the linear coplanarity condition. All models are linked to form a global model. Then their rotation matrices are uniquely decomposed to angular elements. If the object space coordinate system is given, the transformation parameters are also automatically estimated.

The procedure realizes photogrammetry with/without control points for semi-automatic orientation and camera calibration while keeping the user assistance to a minimum. It is very useful for digital-image-based plotters (digital plotters), which feature easy manipulation for everybody who is not familiar with photogrammetry.

TABLE 2. APPROXIMATIONS AND THE MOST PROBABLE VALUES (MPV) OF PARAMETERS OBTAINED IN THE CAMERA CALIBRATION WITHOUT CONTROL POINTS (PART 1).

Interior Orientation Parameters		
	approx.	mpv.
camera dist. (mm)	249.5	249.575 ± 0.016
principal x_0 (mm)	0.0	−0.076 ± 0.019
point y_0 (mm)	0.0	−0.341 ± 0.019
radial lens distort.		
k_1 (mm^{-2})	0.0	$-0.373e^{-7} \pm 0.060e^{-7}$
k_2 (mm^{-4})	0.0	$-0.136e^{-12} \pm 0.298e^{-12}$
tang. lens distort.		
p_1 (mm^{-1})	0.0	$0.609e^{-6} \pm 0.110e^{-6}$
p_2 (mm^{-1})	0.0	$-0.291e^{-6} \pm 0.110e^{-6}$

TABLE 3. APPROXIMATIONS AND THE MOST PROBABLE VALUES (MPV) OF PARAMETERS OBTAINED IN THE CAMERA CALIBRATION WITHOUT CONTROL POINTS (PART 2).

Exterior Orientation Parameters		
photo 1	approx.	mpv.
Ω (deg·dec)	−15.258	−14.952
Φ (deg·dec)	−0.717	−0.182
K (deg·dec)	−88.176	−88.379
X_0 (M)	0.971	0.959
Y_0 (M)	−0.025	−0.255
Z_0 (M)	−0.236	−0.255
photo 10	approx.	mpv.
Ω (deg·dec)	−47.681	−47.327
Φ (deg·dec)	−16.021	−15.588
K (deg·dec)	−97.909	−97.966
X_0 (M)	1.969	1.960
Y_0 (M)	0.495	0.489
Z_0 (M)	−1.139	−1.121

RMS differences of the approximations and the most probable values of target point coordinates in the global model coordinate system=0.797 (mm)

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