Estimating Positional Accuracy of Data Layers within a GIS through Error Propagation

Lawrence V. Stanislawski, Bon A. Dewitt, and Ramesh L. Shrestha

Abstract

The positional accuracy of a GIS layer can be separated into absolute and relative components. Accepted standards for estimating horizontal accuracy in cartographic data quantify absolute positional accuracy only. However, relative accuracy values that describe variability in spatial relationships of coordinate information — such as variance of area, azimuth, and distance computations — can be valuable to research and decision making. This paper presents a technique for quantifying absolute and relative positional accuracy estimated through error propagation from a covariance matrix for affine transformation parameters. This technique was developed and tested with a spatial data set manually digitized from a simulated 1:24,000-scale map whose errors were restricted to those of the electrostatic plotter. A sequence of transformation tests was performed, using from 4 to 40 control points per test. Estimates for combined error associated with electrostatic plotting and manual point-mode digitizing were inversely related to the number of control points up to about 20. Semi-major axes for point certainty regions at a 39.4 percent confidence level ranged from 1.86 to 5.45 metres (0.0775 to 0.227 mm at map scale).

Introduction

Geographic information systems (GIS) are designed for managing, mapping, and analyzing numerous layers of spatial data (Berry, 1987). A layer of spatial data contains a topologically organized set of coordinate pairs that represent features in a real-world coordinate system.

Uses of GIS range from relatively simple tasks of spatial data display to more complex tasks such as the analysis of multi-layered data. From a resource management standpoint, GIS-derived results can impact operational or planning decisions, and the validity of a decision is influenced by the reliability of data from which it is derived (Smith *et al.*, 1991). Thus, prudent use of GIS-supplied information demands an accuracy assessment of all data involved. For a review of some accuracy assessments for spatial data, refer to Goodchild and Gopal (1989) and Gopal and Woodcock (1994).

While GIS data consist of both attribute and positional components, this paper considers only the two-dimensional positional accuracy of GIS data layers. The primary purpose of this paper is to present a technique for estimating absolute and relative positional accuracy of data layers within a vector-based GIS.

Background

Positional accuracy can be separated into two components: absolute and relative (U.S. Department of Interior, 1990). Absolute positional accuracy addresses how closely all positions on a map or data layer match corresponding positions of features they represent on the ground in a desired projection system (i.e., frame of reference). Relative positional accuracy of a map considers how closely all the positions on a map or data layer represent their corresponding geometrical relationships on the ground. In other words, relative positional accuracy reflects the consistency of any position on a map with respect to any other. While absolute positional accuracy of a map may directly influence relative accuracy, limited research has been performed to study this relationship.

Standards have been developed for categorizing positional accuracy in a map. These include the U.S. National Map Accuracy Standards (Thompson, 1979) and accuracy standards for large-scale maps published by the American Society for Photogrammetry and Remote Sensing (ASPRS, 1990; for development of the ASPRS standards, see Merchant (1982), Vonderohe and Chrisman (1985), and Merchant (1987)). These standards provide for quantification of a map's accuracy based on tests that compare map coordinates to ground coordinates determined from an independent check survey of higher accuracy. Results from these standards quantify absolute positional accuracy by comparison with a set of published root-mean-square (RMS) error limits. In addition, the spatial distribution of point errors can be studied with this technique. However, limited insight is gained regarding relative accuracy of geometrical relationships for representative features.

In 1988 the United States Proposed Standard for Digital Cartographic Data was published (ACSM, 1988). This standard was developed to promote efficient coordination between agencies and to avoid data duplication. The United States recently accepted this standard as the national spatial database transfer standard, designated as the Federal Information Processing Standard 173 (FIPS 173) (Moellering, 1993). The four methods that the standard lists for obtaining "measures of positional accuracy" are the following: deductive estimate, internal evidence, comparison to source, and independent source of higher accuracy (ACSM, 1988, pp. 132-133). The standard does not differentiate between absolute and relative positional accuracy. Furthermore, the standard states that reported positional accuracy of a data layer must "consider the quality of the final product after all transformations" (ACSM, 1988, p.132).

Within a spatial information system, transformations to a reference system may be performed with a least-squares adjustment. The method of least squares is based on the assumption that all errors which cannot be explained through

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Surveying and Mapping, Department of Civil Engineering, University of Florida, 345 Weil Hall, Gainesville, FL 32611-6580.

some functional relationship between a set of observations will be random and normally distributed. A transformation model to a geographic reference system is an imperfect functional relationship between unknown parameters and coordinate observations. Imperfection arises from a lack of understanding of all contributing factors, or because the selected model is a simplification of the relationship. This imperfection can be referred to as bias which may produce systematic errors.

Accuracy (i.e., deviation of an estimate from the true value) can be quantified by the mean-squared error which is the sum of the variance and squared bias for an estimator (Mikhail and Gracie, 1981; Dracup, 1993). Least-squares adjustments minimize the mean-squared error of a linear model and utilize a standard normal distribution to estimate the magnitude of errors that are not explained by the prediction model. A transformation model solved through least squares is a statistical relationship from which inferences about the precision and accuracy of the solution can be derived. This paper presents some uses of this statistical relationship with regard to positional accuracy of data within a GIS.

Methodology

Transformation Model

The approach involves error estimates propagated from the variance-covariance matrix of the parameters computed for a six-parameter, weighted affine transformation model. In the test, transformations were made from raw digitizer coordinates directly into the reference system. Parameters and weights were determined through the general least-squares adjustment technique as described by Mikhail and Gracie (1981). In this form, the model is non-linear and can accept full covariance for both sets of coordinates. Absolute positional accuracy is represented with certainty regions for transformed points, and relative accuracy is represented with confidence intervals for distance and azimuth values computed between transformed points. Although not performed in this research, confidence intervals for area can be determined with this method and formulas presented by Griffith (1989). Propagation results were validated by comparisons to probabilities expected from normally distributed errors.

A weighted least-squares adjustment minimizes the sum of squares of the weighted residuals. *A priori* weights for observations are computed by inverting the covariance matrix of the observations. A transformation model in a weighted form is more flexible than an unweighted form, but various solutions for the parameters can be obtained depending on the selected weights. Consequently, the validity of the model depends on the accuracy of the computed weights. For the purposes of this study, the set of defined control coordinate observations was considered to be accurate to its least significant digit. This was the *a priori* and *a posteriori* estimate of the standard deviation for the control. The other set of coordinates, the hand-digitized set, was weighted, *a priori*, with an identity matrix. The *a posteriori* covariance matrix for this set of observations was computed to be equivalent to an identity matrix times the variance of unit weight after convergence. In this way, the variance of unit weight became practically equivalent to one and always passed the chi-square test.

Passing the chi-square test with a sufficiently large sample suggests that the residuals times their associated weights are normally distributed. Knowing this, regions of certainty can be propagated for the transformed points using a bivariate normal distribution. In addition, confidence intervals for quantities—such as distance, azimuth, angle, or area—computed from these points can be estimated through error propagation and the t distribution (Mikhail and Gracie, 1981).

Test Data

Plotting and Digitizing Error

In order to test the transformation and error propagation programs, a grid of 144 points (ticks) with projected polyconic coordinates were generated using ARC/Info. The points covered a 7.5-minute latitude and longitude range, typical of a USGS quadrangle map, and were plotted at a scale of 1:24,000. The ticks had a spacing of 4.8 cm. The polyconic-projected ticks were plotted on paper with a Hewlett-Packard 7600 Series electrostatic plotter using a line width of 1/400 inch (0.0635 mm). The ticks were then digitized without repetition using a Calcomp 9100 digitizer having a resolution of 50 lines per mm and an accuracy of $\pm$ 0.254 mm (Calcomp, 1985). The digitizer is not backlit and the tablet plane was tilted about 20 degrees from vertical.

Digitized points were transformed using the weighted affine procedure, 11 separate times, into a metric polyconic system using 4, 6, 8, 10, 14, 17, 21, 25, 30, 35, and 40 common control points that were distributed symmetrically and mostly at the perimeter. Up to 112 of the remaining points were utilized as check points. Thus, all data sets had 112 check points except the sets having 35 and 40 control points which had 109 and 104 check points, respectively.

Polyconic coordinates projected by the ARC/Info software were computed to the 0.00001-m decimal place, so a variance of 10^{-10} m^2 was used for the control coordinates. Variance for each digitizer coordinate was set to one and multiplied by the variance of unit weight after convergence.

Error Propagation

Point error ellipses, and standard deviations for distances and azimuths, were determined by the general law of propagation of variances and covariances for the nonlinear case (Mikhail and Gracie, 1981). This method utilizes the Jacobian matrix for the affine equations, J_a, the covariance matrix of the affine parameters, Σ_a, and the variances for the unknown coordinates to compute a covariance matrix for the transformed coordinates. The variances of unknown coordinates are appended to the covariance matrix of affine parameters to form Σ_a', and the covariance matrix of transformed coordinates, Σ_{tc}, is computed as follows:

$$\Sigma_{tc} = J_a \Sigma_a' J_a^T.$$

Error ellipses for transformed check points were generated at the 0.394, 0.500, 0.865, 0.900, and 0.998 probability levels using a bivariate normal distribution with two degrees of freedom (Mikhail and Gracie, 1981). These propagated error ellipses can be referred to as certainty regions at corresponding confidence level percentages (Figure 1). To validate these computed certainty regions, expected confidence levels

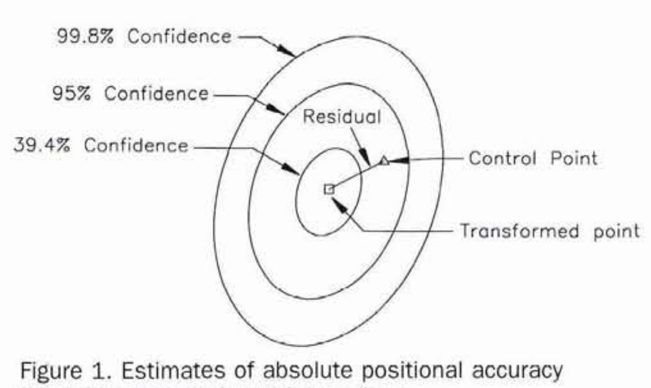

Figure 1. Estimates of absolute positional accuracy through propagated certainty regions.

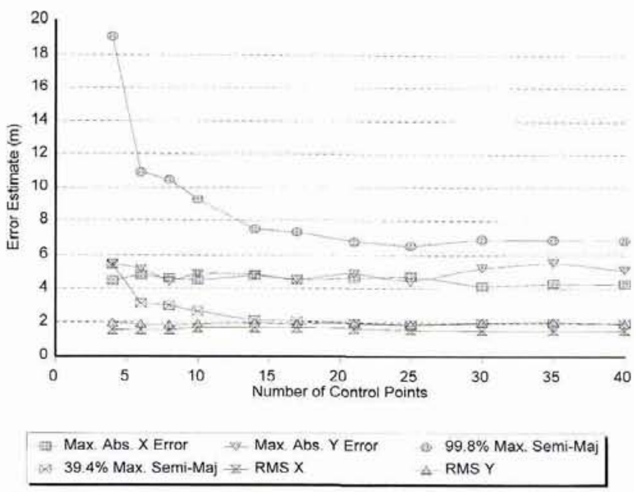

Figure 2. Statistics for transformed check points: propagated semi-major axes for the 39.4 and 99.8 percent confidence regions, root-mean-square error, and maximum absolute errors for the transformed X and Y coordinates.

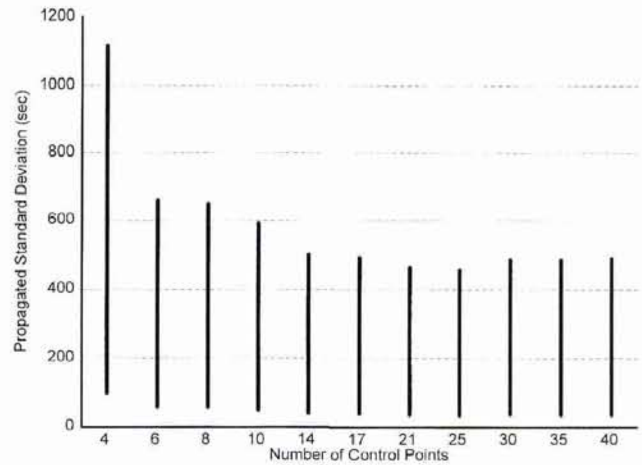

Figure 4. Propagated maximum and minimum standard deviations for azimuths computed between transformed check points.

were compared to the percentage of check points that fell within their associated transformed point's error ellipse at corresponding levels of confidence.

Standard deviations were propagated for distances and azimuths between transformed check points. All possible distances were computed between check points, which provided up to 6216 check distances ranging from 2.21 km to 18.45 km. One-sigma (68 percent) and two-sigma (95 percent) confidence intervals based on the t distribution were computed for each transformed distance and azimuth. The percentage of control distances and azimuths that fell within these confidence intervals was determined.

Results and Discussion

A posteriori standard deviations for digitizer coordinates that were estimated from the weighted least-squares adjustments ranged from 0.0716 mm for 25 control points to 0.1755 mm for four control points. Standard deviations estimated from 30, 35, and 40 control points were 0.0767 mm, 0.0770 mm, and 0.0772 mm, respectively. These estimates are about 4 to 9 times the resolution of the digitizer, which is consistent with digitizing error estimates reported in the literature (Burroughs, 1986; Bolstad *et al.*, 1990).

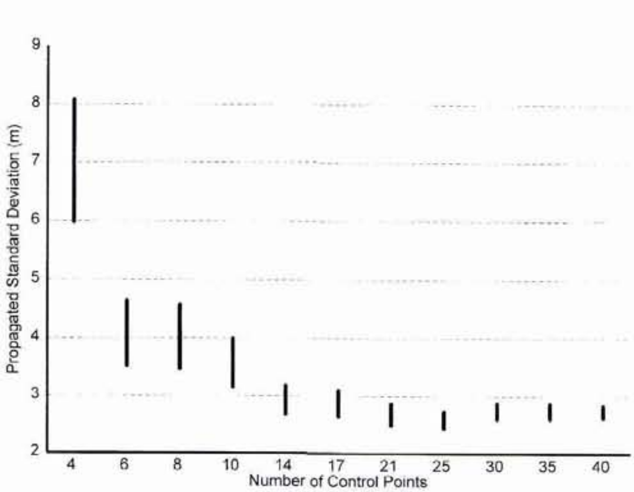

Figure 3. Propagated maximum and minimum standard deviations for distances computed between transformed check points.

Figure 2 illustrates that the size of point error ellipses for transformed points are inversely related to the number of control points. It also shows that semi-major axes for propagated error ellipses are more conservative estimates for accuracy than is the RMS error. In addition, the maximum absolute error for check points is contained within the 99.8 percent certainty region, which is not true for RMS errors.

Propagated standard deviations for distances and azimuths are summarized in Figures 3 and 4, respectively. These figures illustrate that computed quantities become more precise (inverse of standard deviation), and propagated errors for these quantities become more consistent between observations as the number of control observations increases up to about 20. Thus, over the range of control tested, computed quantities are more reliable with increasing control up to about 20 points. Beyond this number of control points, precision does not consistently increase.

Validation of certainty regions and confidence intervals is presented in Figures 5 and 6. The percentages of check points falling within certainty regions approaches a minimum when about 20 control points are used for the transformation (Figure 5). Although the percentage of check points falling within the certainty regions is slightly higher than that expected due to normally distributed errors, using more than 20 control points does not appear to consistently change the percentages. Propagated confidence intervals for distances computed between transformed points show similar validation results (Figure 6). These data suggest that the propagated errors are realistic with respect to statistical theory but slightly conservative.

Jessip (1991) suggests that ten control points are sufficient to model global systematic errors using an unweighted affine transformation, and an additional 20 points should be used to assess accuracy. Further research is necessary to compare the effects of various weighting schemes on propagation results. In this study, unknown coordinate observations were equally weighted and no covariance was imposed

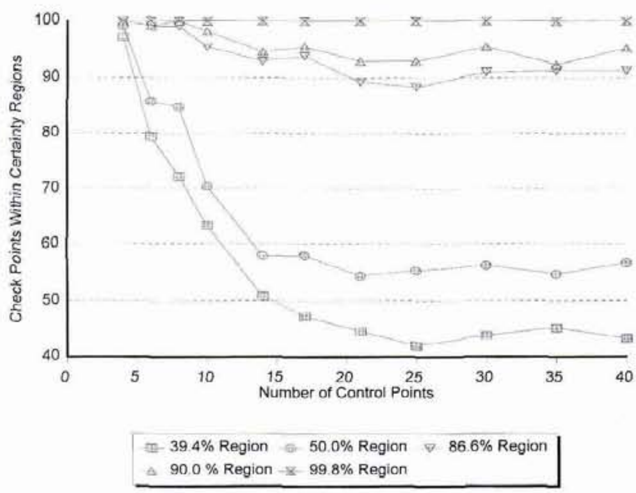

Figure 5. Percentage of check points falling within propagated certainty regions for the transformed points.

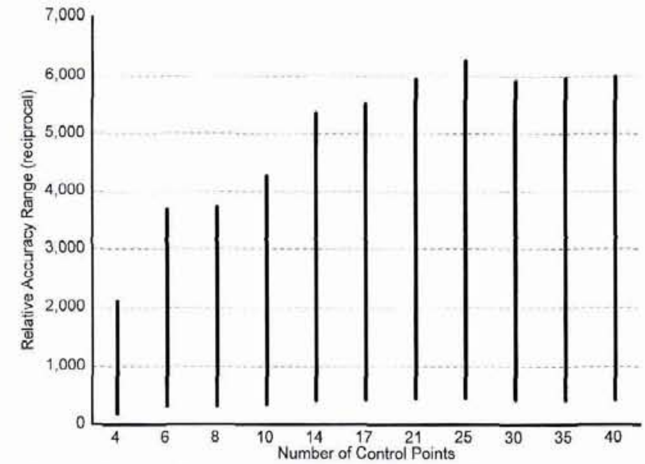

Figure 7. Relative accuracy estimates for the transformed points computed as propagated standard deviation over distance, based on the check distances.

between observations. Aside from studying effects of covariance on propagation results, it is possible to perform robust modifications to weights if a blunder is detected. Robustly modified weights act to counterbalance residuals and ensure that no observation has too great an influence on the solution. Subsequent observation weights or, conversely, residuals could serve as an error distribution map for a data layer. However, blunder detection requires a sufficiently large sample of observations.

The method of quantifying relative accuracy in this paper is similar to the method applied by the Federal Geodetic Control Committee for distance accuracy standards of horizontal control networks (FGCC, 1984). FGCC standards state that "a distance accuracy, 1:a, is computed from a minimally constrained, correctly weighted, least-squares adjustment by

$$a = d/s$$

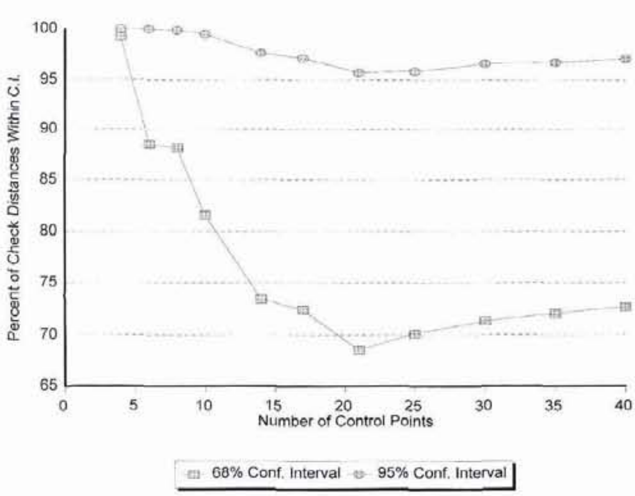

Figure 6. Percentage of check distances falling within confidence intervals propagated for distances computed between transformed check points.

where a is the distance accuracy denominator, s is the propagated standard deviation of distance between survey points obtained from the least-squares adjustment, and d is the distance between survey points."

The least-squares adjustments performed for this research may be considered correctly weighted, but the concept of minimal constraint does not apply in this context. Figure 7 illustrates relative accuracy computed for the range of distances between transformed check points with respect to the number of control points used in each adjustment. For example, in the case of four control points, the shortest distance had a precision of 1/200 while the largest had a precision of 1/2100. Overall, shorter distances had relatively larger propagated standard deviations than longer distances.

When performing a mapping project, it may be necessary to discern the shortest line that can be defined to a specific level of precision. Relative accuracy supplies an answer to this question. For instance, lines shorter than 2 km on the map digitized for this study do not meet 1:1000 precision requirements, regardless of the number of control points used.

The foregoing methodology for accuracy testing is designed to be an integral part of the transformation process. The technique is also applicable as a test of accuracy for a data layer that has an existing (*a priori*) set of geographic coordinates. The control for such a case would be an independent survey of higher accuracy (e.g., an accurate GPS survey). In this case, the transformation could improve the accuracy of the data layer by correcting for existing systematic distortion. If the absolute accuracy of the *a priori* coordinates is to be determined, error ellipse parameters are not appropriate for accuracy estimation because they concern transformed coordinates. For this situation the computed affine parameters such as the x and y translation terms can give some insight as to the accuracy of the *a priori* coordinates.

Conclusion

Geographic information systems are powerful tools for analyzing spatial relationships for a multitude of applications. Consistently testing positional accuracy of data layers in a GIS ensures the reliability of locational information which helps to validate decisions. This paper discussed the use of a weighted least-squares transformation of a GIS data layer and extracting statistical quantities relevant to positional accuracy through error propagation. The transformation may be performed as a part of data layer creation or as an accuracy assessment.

This research was performed to exemplify and test the methodology when applying the affine model; however, the method can be applied to other models as well. The affine model is not necessarily the best for all cases, but it is easy to understand, widely used, and has been shown to reduce global systematic errors from various data sources.

Results suggest that, as the number of control points approaches 20, propagated levels of certainty approach the levels expected from normally distributed errors. Transforming with fewer than 20 points underestimates the accuracy of the data and reduces blunder detection capabilities. Additional research is warranted to validate these results and apply the method on real-world data.

Overall, this technique represents a consistent way to compare the absolute and relative positional accuracy of data layers within a vector-based GIS. The concept could be considered as an extension to current map accuracy testing techniques because it follows an established methodology, that of the Federal Geodetic Control Committee in estimating relative accuracy of geodetic control networks.

References

American Congress on Surveying and Mapping (ACSM), 1988. The proposed standard for digital cartographic data, *The American Cartographer*, 15:9–140.

American Society for Photogrammetry and Remote Sensing (ASPRS), 1990. ASPRS accuracy standards for large-scale maps, *Photogrammetric Engineering & Remote Sensing*, 56(7):1068–1070.

Berry, J.K., 1987. Computer-assisted map analysis: potential and pitfalls, *Photogrammetric Engineering & Remote Sensing*, 53(10): 1405–1410.

Bolstad, P.V., P. Gessler, and T.M. Lillesand, 1990. A variance components analysis of manually digitized map data, *Surveying and Land Info. Systems*, 50(3):201–207.

Burroughs, P.A., 1986. *Principles of Geographical Information Systems for Land Resources Assessment*, Clarendon Press, Oxford.

Calcomp, 1985. *Calcomp 9100 Series Digitizer Operator's Manual 502861-1* (Revision A), Calcomp Group of Sanders Associates, Inc.

Dracup, J.F., 1993. Accuracy estimates from the least-squares adjustment of traverse by condition equations, *Surveying and Land Info. Systems*, 53(1):11–18.

Federal Geodetic Control Committee (FGCC), 1984. *Standards and Specifications for Geodetic Control Networks*, Federal Geodetic Control Committee GPS-003-021-00031-9, Rockville, Maryland, 29 p.

Griffith, D., 1989. Distance calculations and errors in geographic databases, *Accuracy of Spatial Databases* (M. Goodchild and S. Gopal, editors), Taylor & Francis, New York, p. 81–90.

Goodchild, M., and S. Gopal, 1989. *Accuracy of Spatial Databases*, Taylor & Francis, New York, 290 p.

Gopal, S., and C. Woodcock, 1994. Theory and methods for accuracy assessment of thematic maps using fuzzy sets, *Photogrammetric Engineering & Remote Sensing*, 60(2):181–188.

Jessip, M.B., 1991. Systematic horizontal adjustment of positional error in digital line graphs, *Cartography and GIS/LIS*, Technical Paper, ACSM-ASPRS Annual Convention, Baltimore, Maryland, 2:130–140.

Merchant, D.C., 1982. Spatial accuracy standards for large scale maps, *Proc. ACSM 42nd Annual Meeting*, pp. 222–231.

———, 1987. Spatial accuracy specification for large scale topographic maps, *Photogrammetric Engineering & Remote Sensing*, 53(7):958–961.

Mikhail, E.M., and G. Gracie, 1981. *Analysis and Adjustment of Survey Measurements*, Van Nostrand Reinhold Company, Atlanta, 340 p.

Moellering, H., 1993. Life after SDTS: Continuing research needs, *Annual Convention and Exposition Technical Papers*, New Orleans, 15-18 February, pp. 333–340.

Smith, J.L., S.P. Prisley, and R.C. Weih, 1991. Considering the effect of spatial data variability on the outcomes of forest management decisions, *GIS/LIS '91 Proceedings*, Atlanta, Georgia, 1:286–292.

Thompson, M.M., 1979. *Maps for America*, Cartographic Product of the U.S. Geological Survey and others.

United States Department of Interior, 1990. *A Study of Land Information*, prepared by U.S. Dept. Interior in accordance with Public Law 100-409, Nov. 1990.

Vonderohe, A.P., and N.R. Chrisman, 1985. Tests to establish the quality of digital cartographic data: some examples from the Dane County Land Records Project, *Proceedings AUTO-CARTO 7*, pp. 552–559.

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