Using Theoretical Intensity Values as Unknowns in Multiple-Patch Least-Squares Matching

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Abstract

Since its introduction to the photogrammetric community, Least-Squares Matching (LSM) has become a common technique for obtaining accurate location of corresponding points. On several occasions, the basic model was generalized to handle more than two image patches by introducing unknown theoretical intensity values into the mathematical model. With these additional unknowns, gray values rather than gray-value differences are used as observations. In this paper, two aspects of introducing such unknowns are discussed. The first aspect is the equivalence of the generalized model to the basic one. It is shown that, when the generalized model handles only two image patches, the results of the matching are identical to those obtained by the basic approach. The second aspect is the efficiency of the solution. A reduced set of equations is used for the matching, and an efficient way for calculating the unknown theoretical intensity values is derived. Finally, experimental results are presented and discussed.

Introduction

Least-Squares Matching (LSM) was introduced to the photogrammetric community in the early 1980s (e.g., Ackermann, 1984). Since then it has become a common technique for obtaining accurate locations of corresponding image points, provided that good approximations are available. In its basic form, LSM is aimed at finding a certain geometric (and possibly radiometric) transformation between two image patches. The parameters of the transformation are solved for by a least-squares adjustment. The actual "observations" of the adjustment, in the basic (linearized) mathematical model, are gray-value differences between corresponding pixels of the image patches.

In many applications, such as image motion, close-range photogrammetry, and aerotriangulation, it is required to match more than two image patches simultaneously. Several researchers addressed this problem (see, e.g., Agouris (1992), Gruen and Baltsavias (1988), Heipke (1992), Helava (1988), and Wrobel (1988)). Agouris (1992) and Gruen and Baltsavias (1988) extended the model to more than two images by applying geometric constraints to the equation system, while gray-value differences still served as the observations. Heipke (1992), Helava (1988), and Wrobel (1988), in a technique known as object-space LSM, considered the "true" intensities of the ground elements as additional unknowns. This paper will discuss two aspects of adding such unknowns to the mathematical model.

In the idea presented here, the special case of two images, which is used in the classical LSM algorithm, is generalized to handle any number of patches. While gray-value differences can be easily used as observations in the basic mathematical model, it is not straightforward in the case of multiple-patch matching. If we consider, for example, the case of three patches, three gray-value differences can be calculated for each pixel, leading to three observation equations. Obviously, only two observations contribute new information. From two equations selected out of these three (say, the ones with the differences between the first and the second, and the first and the third image patches), each contains a different set of unknowns. Therefore, the entire system is separable to two sets of equations. The practical meaning of this determination is that additional information, hidden in the fact that all the patches are originated in the same source, is not used for the solution.

In order to circumvent this problem, the observations in multiple-patch matching should be gray values. This requires the introduction of unknown theoretical intensity values into the adjustment. Each of these intensity values is associated with its corresponding pixels from the overlapping image patches. In the context of object-space LSM, the theoretical intensity values have the intuitive meaning of the "true" intensity values of the ground at a certain point.

In the next two sections a mathematical model for multiple-patch matching is presented, and a reduced model is derived in order to minimize computation time. The two following sections describe two aspects of using gray values rather than gray-value differences for the LSM. The first aspect is the equivalence between this model and the basic LSM algorithm. It will be shown that, if only two image patches were used, identical solutions would be obtained from both representations. The second aspect is an efficient, simple way to calculate the unknown theoretical intensity values once the geometric transformation is solved. Finally, experimental results are presented and discussed.

Definition of the Mathematical Model

Assume there is a set of ℓ image patches that approximately describe the same area in the object space. Let the patches have the same size, n by m pixels. If there were neither geometric nor radiometric differences between the image patches, and the patches were centered on the same point, they would be identical and therefore the following identity holds for each pixel in each image patch in the set:

$$g^i(r, c) - g^t(r, c) = 0 \tag{1}$$

where $g^i(r, c)$ is the observed gray value of a pixel in row r and column c of patch i and $g^t(r, c)$ is the unknown theoretical intensity value at the corresponding location. If there were radiometric differences between the image patches, the right-hand side of Equation 1 would not equal zero even if there

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were no geometric differences. Rather, it would be equal to a certain error value $e(r, c)$.

In reality, there are also geometric differences which should be modeled by the LSM. Equation 1 is then rewritten as

$$g^i(T^i[r, c]) - g^t(r, c) = e(r, c) \quad (2)$$

where $T^i[r, c]$ is a certain geometric transformation on the pixel coordinates that models the differences. Although other geometric transformations may be introduced, the following derivations assume that the geometric transformation between the patches contains only two shifts. The applicability of this type of transformation is explained by Krupnik (1994). Replacing the general transformation T^i in Equation 2 with the particular transformation of shifts only yields

$$g^i(r + \Delta r^i, c + \Delta c^i) - g^t(r,c) = e(r,c) \quad (3)$$

where Δr^i and Δc^i are the shifts of patch i that are determined by the adjustment. Linearizing Equation 3 to the standard form of an observation equation gives

$$g_0^t(r,c) - g^i(r,c) = g_r^i(r,c)\,\Delta r^i + g_c^i(r,c)\,\Delta c^i - \Delta g^t(r,c) \quad (4)$$

where $g_0^t(r,c) + \Delta g^t(r,c)$ is a theoretical intensity value at location (r,c) (separated to an approximate and unknown parts), and $g_r^i(r,c)$ and $g_c^i(r,c)$ are gray-value gradients across and along the rows, respectively.

Equation 4 will be the same for all the image patches but one. In order to avoid an underdetermined equation system, which yields an infinite number of (yet correct) solutions, two of the parameters should be constrained. A common way to do that is to fix one of the image patches. An observation equations for a pixel of this patch is reduced to

$$g_0^t(r,c) - g^i(r,c) = -\Delta g^t(r,c). \quad (5)$$

Note that fixing one patch to its original location is required only to prevent the model from being underdetermined. The gray values of the constrained patch are still considered as observations and their contribution to the unknown theoretical intensity values is the same as the contribution of any other patch.

The system presented in Equations 4 and 5 solves $n \cdot m$ theoretical intensity values and $2(\ell - 1)$ shifts. Each image patch contributes $n \cdot m$ equations, which brings the total number of equations to $\ell \cdot n \cdot m$. The redundancy of this model is always sufficient. If only two image patches were used, the number of unknowns would be $n \cdot m + 2$ and the number of equations would be $2 \cdot n \cdot m$. The redundancy is therefore $n \cdot m - 2$. In traditional LSM with shifts only, the number of equations is $n \cdot m$ and there are two unknown shifts. The redundancy in both cases is therefore exactly the same.

Equations 4 and 5 can be written in a matrix form: i.e.,

$$\mathbf{y} = \mathbf{A}\boldsymbol{\xi} + \mathbf{e} \quad (6)$$

where $\mathbf{A}$ is the design matrix, $\mathbf{y}$ is the observations vector, $\boldsymbol{\xi}$ is the vector of unknowns, and $\mathbf{e}$ is the error vector. The unknowns are estimated by solving the normal equations

$$(\mathbf{A}^T \mathbf{A})\,\hat{\boldsymbol{\xi}} = \mathbf{A}^T \mathbf{y}\,. \quad (7)$$

The solution is obtained iteratively due to the linearization of the observation equations.

Using a Reduced Equation System

The normal equations system presented in the previous section is relatively large, especially when large image patches are used. Such large image patches are used, for example, to increase matching reliability (see Krupnik (1994)). In such cases computation time becomes significantly long.

An analysis of the model shows, however, that dividing the model into two sets of unknowns, namely, the shifts and the theoretical intensity values, leads to a reduced set of equations. A similar technique is used in solutions of other problems, among which are adjustment of aerotriangulation blocks (e.g., Kraus (1993)) or other implementations of the multiple-patch matching (e.g., Ebner *et al.* (1993)). The theoretical intensity values are calculated once the geometric unknowns (shifts in the case discussed here) are obtained.

The divided model, derived from the original model (Equation 6) is

$$\mathbf{y} = [\,\mathbf{A}_1\ \mathbf{A}_2\,]\begin{bmatrix}\boldsymbol{\xi}_1\\ \boldsymbol{\xi}_2\end{bmatrix} + \mathbf{e} \quad (8)$$

with the design matrix $\mathbf{A}$ split horizontally and the unknown vector split vertically. $\mathbf{A}_1$ and $\mathbf{A}_2$ correspond to the coefficients of the unknown shifts of the patches and the coefficients of the unknown theoretical intensities and contain $2(\ell\text{-}1)$ and $n \cdot m$ columns, respectively. $\boldsymbol{\xi}_1$ and $\boldsymbol{\xi}_2$ correspond to the $2(\ell\text{-}1)$ shifts and $n \cdot m$ theoretical intensities, respectively.

The normal equation system, which solves for the unknowns, is split and has the following form:

$$\begin{bmatrix}\mathbf{N}_{1,1} & \mathbf{N}_{1,2}\\ \mathbf{N}_{2,1} & \mathbf{N}_{2,2}\end{bmatrix}\begin{bmatrix}\boldsymbol{\xi}_1\\ \boldsymbol{\xi}_2\end{bmatrix} = \begin{bmatrix}\mathbf{c}_1\\ \mathbf{c}_2\end{bmatrix} \quad (9)$$

where $\mathbf{N}_{i,j} \doteq \mathbf{A}_i^T \mathbf{A}_j$ and $\mathbf{c}_i \doteq \mathbf{A}_i^T \mathbf{y}$. Solving for $\boldsymbol{\xi}_2$ in the second equation of Equation 9 and substituting in the first leads to the estimation of ξ_1 from

$$(\mathbf{N}_{1,1} - \mathbf{N}_{1,2}\,\mathbf{N}_{2,2}^{-1}\,\mathbf{N}_{2,1})\hat{\boldsymbol{\xi}}_1 = (\mathbf{c}_1 - \mathbf{N}_{1,2}\,\mathbf{N}_{2,2}^{-1}\,\mathbf{c}_2)\,\cdot \quad (10)$$

It can be easily shown (see Krupnik (1994)) that $\mathbf{N}_{2,2}$ is an identity matrix, multiplied by the number of overlapping image patches. Therefore, its inverse exists and is easily obtained. The size of the reduced equation system to be solved is $2(\ell\text{-}1)$, which is significantly smaller than the original system.

The theoretical intensity values, $\boldsymbol{\xi}_2$, can then be estimated by

$$\mathbf{N}_{2,2}\,\hat{\boldsymbol{\xi}}_2 = (\mathbf{c}_2 - \mathbf{N}_{2,1}\,\hat{\boldsymbol{\xi}}_1)\,, \quad (11)$$

or more efficiently as shown later in the paper.

The Equivalence between Classical LSM and Multiple-Patch Matching Models

In this section, the equivalence between the proposed mathematical model and the classical LSM will be shown. Starting from Equation 10, which is the reduced normal equation system for calculating the geometric parameters, the derivations are rewritten for only two image patches. It can be easily shown by simple matrix manipulations that

$$\mathbf{N}_{1,1} - \mathbf{N}_{1,2}\,\mathbf{N}_{2,2}^{-1}\,\mathbf{N}_{2,1} = -\frac{1}{\ell}\begin{bmatrix}(1-\ell)\,\mathbf{A}_{1,1}^T\,\mathbf{A}_{1,1} & \cdots & \mathbf{A}_{1,1}^T\,\mathbf{A}_{1,\ell-1}\\ \vdots & & \vdots\\ \mathbf{A}_{1,\ell-1}^T\,\mathbf{A}_{1,1} & \cdots & (1-\ell)\,\mathbf{A}_{1,\ell-1}^T\,\mathbf{A}_{1,\ell-1}\end{bmatrix} \quad (12)$$

where

$$\mathbf{A}_{1,i}^{T}\,\mathbf{A}_{1,i} = \begin{bmatrix} \sum_{r=0}^{n-1}\sum_{c=0}^{m-1} g_r^i(r,c)\, g_r^i(r,c) & \sum_{r=0}^{n-1}\sum_{c=0}^{m-1} g_r^i(r,c)\, g_c^i(r,c) \\ \sum_{r=0}^{n-1}\sum_{c=0}^{m-1} g_c^i(r,c)\, g_r^i(r,c) & \sum_{r=0}^{n-1}\sum_{c=0}^{m-1} g_c^i(r,c)\, g_c^i(r,c) \end{bmatrix} \tag{13}$$

and

$$\mathbf{c}_1 - \mathbf{N}_{1,2}\,\mathbf{N}_{2,2}^{-1}\,\mathbf{c}_2 = [\mathbf{S}_1^1 \ldots \mathbf{S}_1^{\ell-1}]^T - \frac{1}{\ell}\,[\mathbf{S}_2^1 \ldots \mathbf{S}_2^{\ell-1}]^T \tag{14}$$

where

$$\mathbf{S}_1^i \doteq \mathbf{A}_{1,i}^T\,\mathbf{y}_i = \begin{bmatrix} \sum_{r=0}^{n-1}\sum_{c=0}^{m-1} g_r^i(r,c)\,[g_0^t(r,c) - g^i(r,c)] \\ \sum_{r=0}^{n-1}\sum_{c=0}^{m-1} g_c^i(r,c)\,[g_0^t(r,c) - g^i(r,c)] \end{bmatrix};$$

$$\mathbf{S}_2^i \doteq \begin{bmatrix} \sum_{r=0}^{n-1}\sum_{c=0}^{m-1} g_r^i(r,c) \sum_{i=0}^{l-1} [g_0^t(r,c) - g^i(r,c)] \\ \sum_{r=0}^{n-1}\sum_{c=0}^{m-1} g_c^i(r,c) \sum_{i=0}^{l-1} [g_0^t(r,c) - g^i(r,c)] \end{bmatrix}. \tag{15}$$

For only two image patches, and assuming that the theoretical intensity values $g^t(r, c)$ are approximated by $(g^0(r, c) + g^1(r, c))/2$, Equations 12 and 14 are reduced to

$$\overline{\mathbf{N}} \doteq \mathbf{N}_{1,1} - \mathbf{N}_{1,2}\,\mathbf{N}_{2,2}^{-1}\,\mathbf{N}_{2,1} \tag{16}$$

$$= \frac{1}{2}\begin{bmatrix} \sum_{r=0}^{n-1}\sum_{c=0}^{m-1} g_r^0(r,c)\, g_r^1(r,c) & \sum_{r=0}^{n-1}\sum_{c=0}^{m-1} g_r^0(r,c)\, g_c^1(r,c) \\ \sum_{r=0}^{n-1}\sum_{c=0}^{m-1} g_c^0(r,c)\, g_r^1(r,c) & \sum_{r=0}^{n-1}\sum_{c=0}^{m-1} g_c^0(r,c)\, g_c^1(r,c) \end{bmatrix}$$

and

$$\overline{\mathbf{c}} \doteq \mathbf{c}_1 - \mathbf{N}_{1,2}\,\mathbf{N}_{2,2}^{-1}\,\mathbf{c}_2 = \frac{1}{2}\begin{bmatrix} \sum_{r=0}^{n-1}\sum_{c=0}^{m-1} g_r^1(r,c)\,(g^0(r,c) - g^1(r,c)) \\ \sum_{r=0}^{n-1}\sum_{c=0}^{m-1} g_c^1(r,c)\,(g^0(r,c) - g^1(r,c)) \end{bmatrix}. \tag{17}$$

With traditional LSM, assuming only shift parameters, a linearized observation equation obtaines the following form:

$$g^0(r,c) - g^1(r,c) = g_r^1(r,c)\,\Delta r + g_c^1(r,c)\,\Delta c \tag{18}$$

where $g^0(r, c)$ and $g^1(r, c)$ are the pixel gray values in the "left" and "right" (or "template" and "moving") patch, respectively; $g_r^1(r, c)$ and $g_c^1(r, c)$ are the gray value gradients in the moving patch along and across the rows, respectively; and Δr, Δc are the unknown shift parameters of the moving patch.

Presenting the normal equations for the traditional LSM in their matrix representation yields

$$\overline{\overline{\mathbf{N}}}\,\hat{\xi} = \overline{\overline{\mathbf{c}}} \tag{19}$$

where

$$\overline{\overline{\mathbf{N}}} \doteq \mathbf{A}^T\mathbf{A} = \begin{bmatrix} \sum_{r=0}^{n-1}\sum_{c=0}^{m-1} g_r^0(r,c)\, g_r^1(r,c) & \sum_{r=0}^{n-1}\sum_{c=0}^{m-1} g_r^0(r,c)\, g_c^1(r,c) \\ \sum_{r=0}^{n-1}\sum_{c=0}^{m-1} g_c^0(r,c)\, g_r^1(r,c) & \sum_{r=0}^{n-1}\sum_{c=0}^{m-1} g_c^0(r,c)\, g_c^1(r,c) \end{bmatrix} \tag{20}$$

and

$$\overline{\overline{\mathbf{c}}} \doteq \mathbf{A}^T\mathbf{y} = \begin{bmatrix} \sum_{r=0}^{n-1}\sum_{c=0}^{m-1} g_r^1(r,c)\,(g^0(r,c) - g^{-1}(r,c)) \\ \sum_{r=0}^{n-1}\sum_{c=0}^{m-1} g_c^1(r,c)\,(g^0(r,c) - g^1(r,c)) \end{bmatrix}. \tag{21}$$

It can be clearly observed that

$$\overline{\overline{\mathbf{N}}} = 2\overline{\mathbf{N}} \tag{22}$$

$$\overline{\overline{\mathbf{c}}} = 2\overline{\mathbf{c}} \tag{23}$$

and, therefore,

$$\overline{\overline{\mathbf{N}}}^{-1}\,\overline{\overline{\mathbf{c}}} = \overline{N}^{-1}\,\overline{\mathbf{c}}. \tag{24}$$

This result shows the equivalence between the generalized model of multiple-patch matching presented in this work and the basic formulation of LSM. The advantage of the proposed scheme is its ability to match more than two image patches simultaneously.

Efficient Estimation of the Theoretical Intensities

Because the solution of the adjustment for the multiple-patch matching is obtained iteratively, an estimation for the theoretical intensity values is required in order to improve the approximations for the succeeding iteration. Although it is possible to estimate these values directly from Equation 11, a simpler approach may be taken. Each row of equation system 11 is expanded to the following form:

$$\hat{\Delta g}^t(r,c) = -\frac{1}{\ell}\sum_{i=0}^{\ell-1} (g_0^t(r,c) - g^i(r,c) - g_r^i(r,c)\,\hat{\Delta r}^i - g_c^i(r,c)\,\hat{\Delta c}^i), \tag{25}$$

where the hat mark denotes the estimated values. Rewriting the equation in the nonlinear form:

$$\hat{\Delta g}^t(r,c) = -g_0^t(r,c) + \frac{1}{\ell}\sum_{i=0}^{\ell-1} g^i(r + \hat{\Delta r}^i,\ c + \hat{\Delta c}^i) \tag{26}$$

and, therefore,

$$g_0^t(r,c) + \hat{\Delta g}^t(r,c) = \frac{1}{\ell}\sum_{i=0}^{\ell-1} g^i(r + \hat{\Delta r}^i,\ c + \hat{\Delta c}^i). \tag{27}$$

In other words, the estimation for the unknown theoretical intensity values is obtained simply by averaging the corresponding pixels from all image patches, after resampling them according to the estimated geometric transformation parameters. In any case, this resampling is done for the succeeding iteration of the adjustment procedure and, therefore, the cost of calculating estimations for the theoretical intensity values is minimal.

The latter determination can also be explained intuitively. The solution for the geometric unknowns ensures that

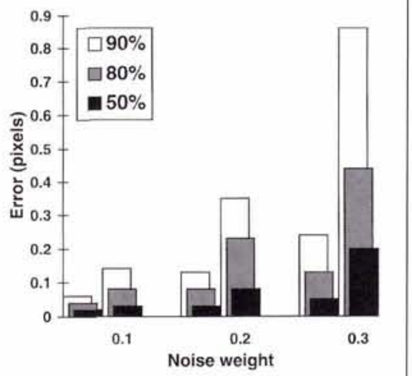

Figure 1. Effects of noise on the quality of matching results.

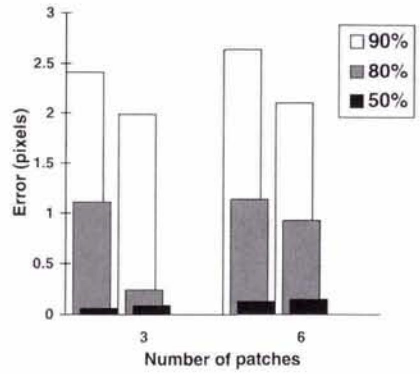

Figure 3. Differences between results obtained by multiple-patch matching and results obtained by matching in pairs.

the new image patches are resampled such that the differences from the theoretical intensity values are minimal. Given a set of gray values of corresponding pixels on different image patches, the value which minimizes these differences is merely their average.

Experimental Results

The idea of multiple-patch matching is demonstrated in this section by a set of experiments that tested the behavior of the matching procedure. Thirty patches from a real image were used, taken from areas with different texture complexities.

Each patch was duplicated (up to six copies were used) and corrupted by random noise according to the following: each pixel was assigned a new value g_n: i.e.,

$$g_n = (1 - w)\, g_0 + wu \tag{28}$$

where w is a noise weight value, g_o is the gray-value of the original image patch, and u is a random number in the range of 0 to 255. Weight values between 0 and 0.3 were tested. One should realize that, when $w = 0.3$, the corrupted patch is significantly different from the original one.

The approximations provided to the matching process were shifted from the centers of the image patches by a few pixels. This was done to check the ability of the procedure to converge to the correct location.

Figure 1 shows the effect of the noise added to the patches on the quality of the results when correct locations

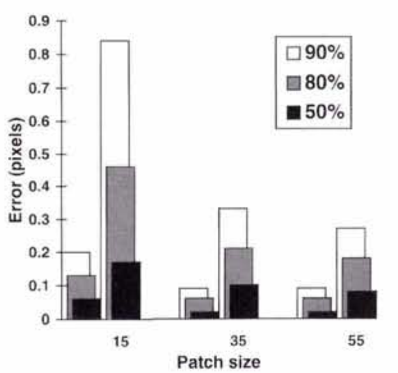

Figure 2. Effects of patch size on the quality of matching results.

were provided to the procedure (left set of bars in each pair) and when approximations are two pixels away from the correct locations in a randomly chosen direction (right set of bars). Each set of bars shows the errors for which 50 percent, 80 percent, and 90 percent of the cases were under. It is clearly seen that, even with noise levels of up to 0.2, and with shifts of two pixels, 90 percent of the image patches converged to a location which deviates from the true location by less than 0.35 pixels.

Figure 2 shows the influence of the patch size on the matching results. The left set of bars in each pair refers to a case where the noise weight was set to 0.1 and the shift of the approximations was set to 1 pixel. The respective values for the right sets of bars are 0.2 and 3. The higher values of noise and shifts had an influence on the results for all patch sizes. However, it can be clearly observed that, while with a patch size of 15 by 15 pixels the error for the "90 percent" value in the left set of bars is approximately 9 times larger than that value on the right set, this factor is only 3.5 for the 55 by 55 size patch. As expected, larger image patches yield more reliable results.

Finally, Figure 3 demonstrates the different results obtained by using multiple-patch matching (left set of bars) versus matching in pairs (right set). Here, 25- by 25-pixel patches with a noise weight of 0.2 were used, and approximations were shifted by four pixels. The results (especially the 90 percent and the 80 percent values) indicate that, in the case where matching was performed in pairs, more points converged to wrong locations than in the case where multiple-patch matching was used. This is explained by the following observation. If, for example, one or more of the overlapping patches contain weak signal, or if convergence is marginal, matching in pairs may converge to a wrong location, because it does not use the knowledge that all the patches are actually originated in the same source. Multiple-patch matching improves the robustness of the solution by taking advantage of this knowledge. Therefore, problematic cases that are not solved by matching in pairs are overcome by matching all the patches simultaneously.

Summary

In this paper, two aspects of using unknown theoretical intensity values in multiple-patch least-squares matching were emphasized. The first discussed the equivalence between matching more than two images, using unknown theoretical intensity values, and classical LSM where only two images are matched, and gray-value differences are used as observations. The second aspect is a low-cost solution (in terms of computation time) for the large equation system generated by adding the unknown intensity values.

Multiple-patch matching provides a simultaneous and objective solution for finding corresponding points on more than two images. Although the derivations were shown for a simple model of transformation between the image patches, generalizing it to other transformation is straightforward. The solution is efficient and suitable for any number of image patches, at any desirable size.

The experimental results showed that acceptable results are obtained even when the approximations are two pixels away from the correct locations, and when noise with a weight value of 0.2 is added to each image patch. With high noise levels and relatively bad approximations, the use of large image patches increased the quality of the results significantly. Finally, matching all the image patches simultaneously, using the multiple-patch matching method, yielded better results than matching them in pairs.

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