GPS-Controlled Strip Triangulation Using Geometric Constraints of Man-Made Structures

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Abstract

Conventional block adjustments have been widely used to determine both photogrammetric ground point coordinates and the exterior orientation parameters of photography for mapping purposes. Ground control points are necessary to relate the image coordinate system to the object space coordinate system and to ensure the geometric stability of the conventional photogrammetric block as well as to control error propagation. The major impact of cost and time consumption for ground control establishment on any mapping project is the primary reason that photogrammetrists have been looking for a replacement for ground control by auxiliary data (e.g., Global Positioning System). This paper describes a new technique for GPS-controlled single strip triangulation using geometric constraints of man-made structures (e.g., high voltage towers, high rise buildings) located approximately along the flight line. The effects of the different GPS measurement accuracies were also investigated. Both the precision and the reliability analyses of the GPS bundle strip adjustment with these constraints were carried out on simulated and real data.

Introduction

Since the launch of the Global Positioning System (GPS) satellites in the early 1980s, photogrammetrists have realized the application of GPS for their particular interests (i.e., aerial triangulation). With this technology, the position of the aircraft at the individual exposure moments can be precisely determined. These positions can then be introduced into the combined GPS-photogrammetric block adjustment as weighted observations for the exposure stations, reducing the number of control points to a minimum. Aerial triangulation can even be completed without any control points, provided that the satellite signals are not blocked during the flight mission (Lapine, 1991) and as long as datum transformations are known.

However, there are a few problems that require attention in GPS-assisted aerial triangulation. These are the GPS antenna offset calibration, interpolation of the exposure time, the initial phase ambiguity resolution, signal interruptions, and datum problems (Ackermann, 1992).

After processing of kinematic GPS observations, coordinates of the antenna phase centers are available in the WGS84 reference frame. Most ground coordinates are defined with respect to a national coordinate system (e.g., UTM). The transformation between these coordinate systems can be based on published formulas (Colomina, 1993) or a set of common reference points available in both systems. Elevations are usually related to the ellipsoid and must be corrected for the geoid undulations.

Ground control points (planimetric points along the perimeter of the block and relatively dense chains of vertical points across the block) are mandatory for relating the image coordinate system to the object coordinate system and to ensure the geometric stability of the conventional block. The minimum control requirement for absolute orientation is three non-collinear points. For the GPS-photogrammetric block, this condition is met by using GPS observations at the perspective centers as control information. Because the GPS observations of the exposure stations are almost collinear for strip triangulation, the above condition is not satisfied and, therefore, the roll angle (i.e., around the flight direction) can't be recovered reliably. This makes it necessary to use ground control points to solve for the remaining exterior orientation parameters (Alobaida, 1993). Other authors, for example, Merchant (1974), proposed the Method of Mixed Ranges (MMR) to resolve some of these problems in aerial analytical photogrammetry.

This paper reviews the concept of the GPS observable used in precise photogrammetric applications and describes a new technique for GPS-controlled strip triangulation using geometric constraints of man-made structures (e.g., high voltage towers and high-rise buildings) located approximately along the flight line. The effects of the accuracies of different GPS measurements were also investigated. Precision and reliability analyses were performed on both simulated and real data. All results were obtained using GAP (General Adjustment Program) developed by the first author at the Department of Geomatics Engineering (Digital Photogrammetry Group), The University of Calgary. GAP is an integrated GPS, photogrammetric, and geodetic adjustment program. It can be used to adjust a geodetic network (e.g., distances, directions, azimuths) and photogrammetric block (e.g., image coordinates, exterior orientation parameters) or a combined geodetic/photogrammetric block. It can also be used to incorporate GPS positions of the exposure stations into a block adjustment.

GPS Observables Used in Precise Photogrammetric Applications

Due to the high accuracy required for aerotriangulation, GPS phase measurements are needed to meet this requirement. In order to eliminate the effects of systematic errors introduced by GPS, double-difference GPS phase measurements in differential mode are used. The reason is that most GPS errors affecting GPS accuracy are highly correlated over a certain area and can be eliminated or reduced. The observation equation for DGPS phase measurement can be found in Lachapelle *et al.* (1992). Because atmospheric and orbital errors are gener-

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ally small or negligible for short monitor-remote distances (e.g., less than 10 to 20 km), such a system configuration is used. The satellite and receiver clock errors are eliminated using DGPS method, but the receiver noise is amplified by a factor of two. The phase observable is used extensively in kinematic mode where the initial ambiguity resolution can be achieved using static initialization or "On-the-Fly" methods. Accuracy at the centimetre level can be obtained if cycle slips can be detected and recovered (Cannon, 1990).

GPS-Supported Aerotriangulation

The main purpose of aerial triangulation (AT) is to determine the ground coordinates of a large number of terrain points and the exterior orientation parameters of aerial photographs using as few control points as possible. The best scenario in mapping projects is to have the exterior orientation parameters accurate enough so that the AT can be neglected. The GPS-derived accuracy for attitude parameters is about 15 arc minutes, still far from what could be obtained from a conventional block adjustment. Therefore, aerial triangulation is still one of the important steps in mapping and can not be avoided.

The integration of GPS measurements with photogrammetric blocks allows for the accurate determination of the coordinates of the exposure stations, resulting in a reduction of the number of ground control points to a minimum. The combined adjustment of photogrammetric data and GPS observations can be carried out by introducing GPS observation equations into the conventional block adjustment (Ebadi and Chapman, 1995). Empirical investigations (Frieβ, 1991) showed that, in addition to the high internal accuracy of GPS aircraft positions (σ = 2 cm), drift errors may occur due to ionospheric and tropospheric errors, satellite orbital errors, and the uncertainty of the initial ambiguities. Of these remaining errors, incorrect carrier phase ambiguities are the major contributor of the drift errors to the exposure station position.

GPS-Controlled Strip Triangulation with Geometric Constraints of Man-Made Structures

The inherent geometry of a photogrammetric block and the common tie points in consecutive strips make it possible to recover all three rotation angles in the combined block adjustment. Unfortunately, this method can't be used for a single strip because the GPS coordinates of the exposure stations do not recover the roll angle of the aircraft. As a consequence, control introduced by airborne GPS alone leaves an ill-conditioned, if not singular, system of normal equations. Ground control points can be used along the flight line to overcome this problem.

A new technique for GPS-controlled strip triangulation was developed based on the geometric constraints of man-made structures (e.g., high voltage towers, high-rise buildings) located along the flight line. The observation equations with proper weights for these constraints are introduced into the combined strip adjustment. The constraint observation equations for a high voltage tower or similar structure are written as

$$\begin{bmatrix} X_j \\ Y_j \end{bmatrix} - \begin{bmatrix} X_i \\ Y_i \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{1}$$

where (X_j, Y_j) are the horizontal coordinates of the top of the structure, and (X_i, Y_i) are the horizontal coordinates of the bottom of the structure.

The weights for these equations should be appropriately chosen because the top and the bottom of the structure must have the same horizontal coordinates. However, the absolute ground coordinates of the structure are not required because

Table 1. Simulated Strip

Strip Information	
Number of Photos	50
Photo Scale	1:5000
Focal Length	152 mm
Terrain Elevation Difference	150 m
Average Flying Height	900 m
Forward Overlap	60 percent
Photograph Format	23 cm by 23 cm
Accuracy of Image Coordinates	5 µm
Accuracy of Ground Control Points	0.1 m
Accuracy of GPS	0.25 to 1.0 m
Tower Height	15 m
Number of Towers	50
Number of Pass Points per Photo	15

the top and the bottom of the structure are used in a manner similar to horizontal pass points. The main idea is to use a number of towers along the flight line in order to recover the roll angle of the aircraft and to use these constraints to minimize the number of ground control points.

Precision and Reliability Measures

In order to properly assess the performance of the proposed method, appropriate accuracy measures must be used. With this in mind, we may employ the theory of reliability, as developed by Baarda (1968), to evaluate the quality of adjustment results of the geodetic networks. According to Baarda, the quality of adjustment considers both precision and reliability. Precision evaluation consists of comparing the covariance matrix $\mathbf{C}_X$ of the adjusted coordinates with a given matrix $\mathbf{H}_X$ (criterion matrix). The error ellipsoid derived from $\mathbf{C}_X$ should lie inside the error ellipsoid described by $\mathbf{H}_X$ and should be as similar to $\mathbf{H}_X$ as possible. This approach can be used to check whether or not a required accuracy has been achieved. The covariance matrix of object points, $\mathbf{C}_X = \sigma_0^2 \mathbf{Q}_{XX}$, is generally taken as the measure of theoretical precision. The average theoretical precision of n object points is given as

$$\overline{m} = \sigma_0 \sqrt{\mathrm{tr}(\mathbf{Q}_{xx}/3n}. \tag{2}$$

The average practical precision of object points for simulated or check point data is

$$\overline{\mu} = \sqrt{\Sigma \left(\Delta X^2 + \Delta Y^2 + \Delta Z^2\right)/3n} \tag{3}$$

where ΔX, ΔY, and ΔZ are differences between the adjusted and known coordinates of an object point. Similarly, $\overline{m}_x$, $\overline{m}_y$, $\overline{m}_z$ and $\overline{\mu}_x$, $\overline{\mu}_y$, $\overline{\mu}_z$ are the average theoretical and practical precision of object points in the X, Y, and Z directions, respectively.

Reliability analysis includes internal and external reliabilities. The ability to discover blunders in one particular observation is referred to as internal reliability and the effect of an undetected blunder in the observation on unknown parameters is measured by external reliability. For reliability analysis, the local redundancy numbers, $r_i = (\mathbf{Q}_{vv}\mathbf{P})_{ii}$, need to be determined, where $\mathbf{Q}_{vv}$ is the cofactor matrix of residuals and $\mathbf{P}$ is the weight matrix of observations. A single blunder is generally assumed to exist for the application of this method. The internal and external reliability formulas are given as (Deren and Jie, 1989)

$$\nabla_0 L_i = \sigma_{L_i} \frac{\delta_0}{\sqrt{r_i}} = \sigma_{L_i}, \delta_{0,i} \tag{4}$$

$$\overline{\delta}_{0,i} = \sqrt{1 - r_i} \cdot \delta_{0,i} \tag{5}$$

where σ_{L_i} is the RMSE of the ith observation L_i, δ_0 is the non-

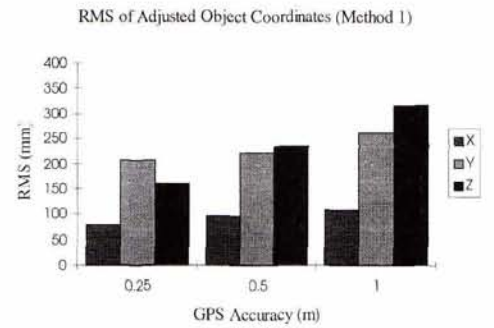

Figure 1. RMSE of X, Y, and Z coordinates of all object points (Method 1).

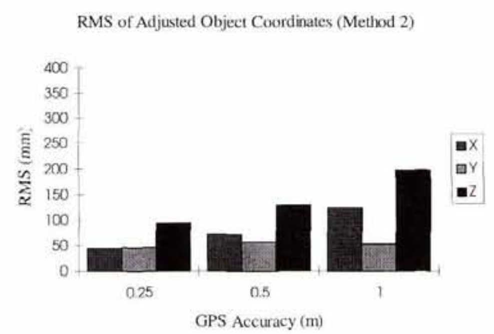

Figure 2. RMSE of X, Y, and Z coordinates of all object points (Method 2).

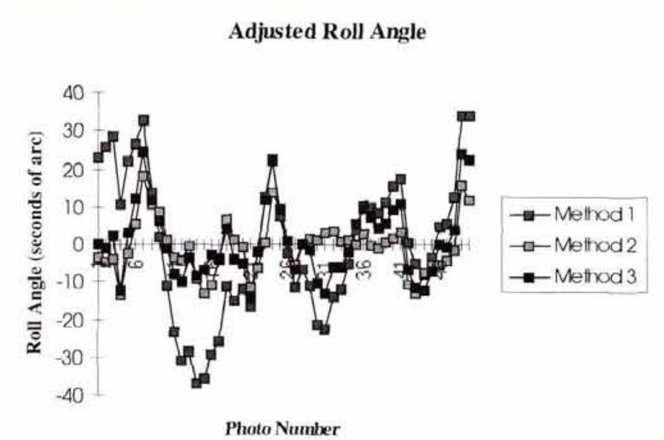

Figure 3. Adjusted roll angles from the three methods.

centrality parameter, $\delta_{0,i}$ is the internal reliability factor of observation L_i, $\nabla_0 L_i$ is the minimum blunder that can be detected statistically, and $\bar{\delta}_{0,i}$ is the external reliability factor of observation L_i. With a confidence level of 99 percent and a power of 93 percent, the non-centrality parameter, δ_0, is then equal to 4.0.

Results with Simulated Data

Many experiments were conducted to evaluate the performance of the new model. Initially, one single strip of 50 photographs was considered. The interior and exterior orientation parameters were predefined. The image coordinates of all object points (pass points and tower points) were computed using the known exterior orientation of each photograph. Table 1 lists the information concerning this simulated strip.

The behavior of the new model was studied under different conditions, such as varying GPS accuracies at the perspective centers. The performance of this technique was evaluated using the standard deviations of the coordinates of pass points obtained from their variance-covariance matrix and comparing the adjusted coordinates with simulated coordinates. The following methods of the strip adjustment were carried out:

- GPS-Photogrammetric strip adjustment with two ground control points and without tower points,
- GPS-Photogrammetric strip adjustment with no ground control points and with tower points, and
- full control strip adjustment (no GPS data).

Figure 1 shows the root-mean-square error (RMSE) of the X, Y, and Z coordinates of all pass points without using tower points and Figure 2 shows the RMSE of the X, Y, and Z coordinates of all pass points including tower points. The RMSE of the X, Y, and Z coordinates of pass points for the full ground control adjustment are 45, 57, and 197 mm, respectively.

Comparing Figures 1 and 2, it is shown that, if GPS can provide accuracy at the level of 0.25 to 0.5 m for camera exposure stations coordinates, the RMSE values for all object coordinates are better than or equal to those obtained from the full ground control version. These results confirm that constraint information from the tower points can replace the ground control points and eliminate the need for the second parallel strip of photography which has been adopted conventionally to improve the geometry of the strip. To see how this technique recovers the roll angle of the aircraft, Figure 3 shows the adjusted roll angle of each photo obtained from the three methods ($\sigma_{GPS} = 0.25$ m). As seen in this figure, the adjusted roll angle recovered from Method 2 (tower points included) is almost the same as that which was obtained from Method 3 (full ground control version).

In the reliability analysis, redundancy numbers, internal reliability factors, and external reliability factors were computed for various observations (image coordinates, GPS coordinates of exposure stations). Tables 2 and 3 show the reliability measures for image coordinates and GPS coordinates of exposure stations for the various methods.

These values are rated as good ($0.5 < r_i$, $\bar{\delta}_{0,i} < 4.0$), acceptable ($0.1 \le r_i < 0.5$, $4.0 \le \bar{\delta}_{0,i} < 10.0$), bad ($0.04 \le r_i < 0.1$, $10.0 \le \bar{\delta}_{0,i} < 20.0$), and not acceptable ($r_i < 0.04$, $20.0 \le \bar{\delta}_{0,i}$), (Förstner, 1985). The best values for the reliability measures have been obtained from Method 2, which implies that including tower points in the GPS-controlled strip triangulation improves the reliability of both image coordinates, especially the y coordinates of image points and GPS observations.

In order to determine the minimum number of towers which are needed to serve as a constraint, GPS single-strip adjustments were carried out for the different number of tower structures. Table 4 shows the RMSE of the check points for the X, Y, and Z coordinates. This table shows that if there is one tower in every five photos, then the same results can be expected as those obtained from the case when there is one tower in each photo. Various trials of GPS single-strip adjustments were executed based on different heights for the tower structures. It was found that the taller the structure, the better the accuracy of the adjustment. Table 5 shows the RMSE of the check points for various trials. It can be concluded from this table that for a typical large scale mapping project if imagery can provide towers with heights in the

Table 2. Reliability Measures for Image Coordinates

Reliability Measure	Method 1		Method 2		Method 3	
	x	y	x	y	x	y
r_i	0.28	0.11	0.34	0.54	0.23	0.39
$\delta_{0,i}$	8.34	12.52	8.41	5.73	9.96	6.61
$\bar{\delta}_{0,i}$	7.21	11.85	7.11	3.93	8.98	5.21

Table 3. Reliability Measures for GPS Coordinates of Exposure Stations

Reliability Measure	Method 1			Method 2		
	X	Y	Z	X	Y	Z
r_i	0.74	0.80	0.84	0.74	0.85	0.82
$\delta_{0,i}$	4.65	4.47	4.36	4.66	4.34	4.42
$\bar{\delta}_{0,i}$	2.36	1.98	1.72	2.37	1.67	1.85

Table 4. RMSE of Check Points

Number of Towers	X (m)	Y (m)	Z (m)
2	0.049	0.203	0.111
3	0.049	0.173	0.104
4	0.042	0.096	0.092
5	0.041	0.085	0.090
6	0.040	0.056	0.091
11	0.043	0.048	0.088
50	0.042	0.044	0.090

Table 5. RMSE of Check Points

Height Of Tower (m)	X (m)	Y (m)	Z (m)
15	0.044	0.046	0.092
30	0.031	0.030	0.091
45	0.019	0.025	0.087
60	0.019	0.024	0.087

Table 6. Project Parameters

Strip Information	
Purpose of Photography: Corridor Mapping, East of Edmonton, Canada	
Number of Photos	22
Number of Strips	2
Photo Scale	1:10,000
Focal Length	153.692 mm
Average Terrain Elevation Difference	50 m
Average Flying Height	1500 m
Forward Overlap	60 percent
Photograph Format	23 cm by 23 cm
Precision of Image Coordinates	5 μm
Precision of Ground Control Points	0.2 m
Precision of GPS	0.25 to 1.0 m
Number of Ground Control Points	14
Number of Check Points	9
Tower Height	40 m
Number of Towers	29
Number of Pass points per Photo	20

range of 2 percent to 7 percent of the flying height, then the GPS single-strip adjustment with constrained tower structures can offer the required accuracy.

Results with Real Data

Encouraging results from the simulated data encouraged us to apply this new technique using real data. Trans Alta Utilities Ltd. conducted a corridor mapping project in 1992, east of Edmonton, Alberta, Canada. The information concerning this project is given in Table 6. A full control block adjustment was executed to estimate the GPS camera exposure stations due to the fact that actual airborne GPS data were not available. Image coordinates of all control and pass points were provided by Trans Alta Utilities Ltd., Calgary, while the image coordinates of tower points were measured using Wild AC1 available at the Department of Geomatics Engineering.

Three scenarios were selected regarding the geometric configurations of control and tower points, i.e.,

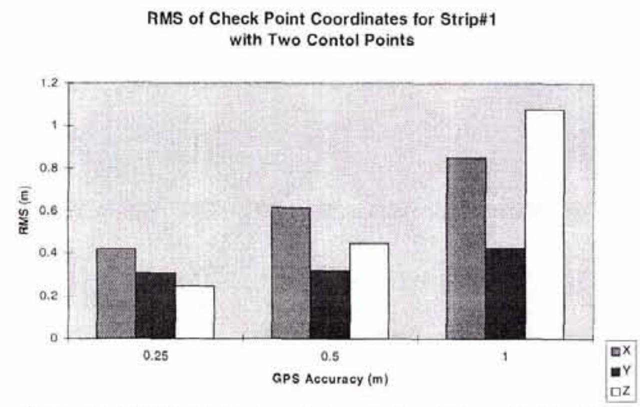

Figure 4. RMSE of check points obtained from Method 1.

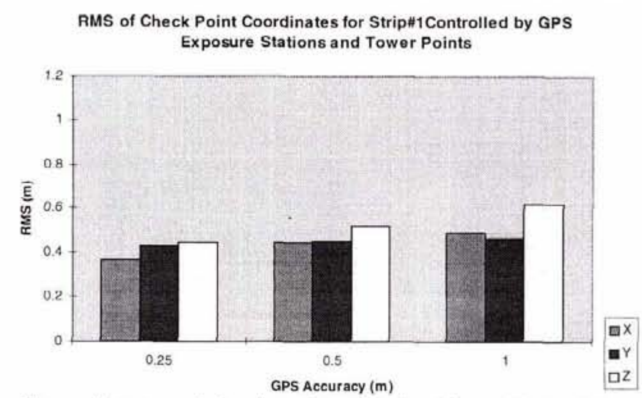

Figure 5. RMSE of check points obtained from Method 2.

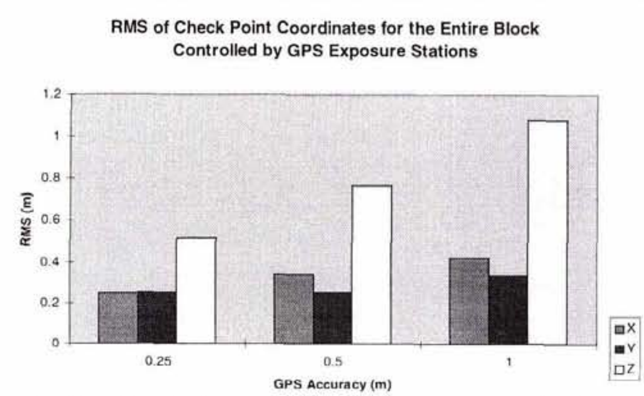

Figure 6. RMSE of check points obtained from Method 3.

Table 7. Reliability Measures for Image Coordinates

Reliability Measure	Method 1		Method 2		Method 3	
	x	y	x	y	x	y
r_i	0.13	0.43	0.22	0.51	0.13	0.43
$\delta_{0,i}$	14.51	6.18	12.08	5.70	14.20	6.23
$\bar{\delta}_{0,i}$	13.97	4.68	11.26	4.00	13.64	6.23

Table 8. Reliability Measures for GPS Coordinates of Exposure Stations

Reliability Measure	Method 1			Method 2			Method 3		
	X	Y	Z	X	Y	Z	X	Y	Z
r_i	0.71	0.64	0.77	0.74	0.69	0.76	0.73	0.67	0.74
$\delta_{0,i}$	4.78	5.03	4.58	4.66	4.84	4.62	4.68	4.88	4.65
$\bar{\delta}_{0,i}$	2.60	3.03	2.19	2.36	2.68	2.28	2.40	2.80	2.37

(1) GPS-photogrammetric strip adjustment with two ground control points,
(2) GPS-photogrammetric strip adjustment with tower points and no control points, and
(3) GPS-photogrammetric block adjustment to be used as a reference.

Figures 4 and 5 show the RMSE values for the check points obtained from Methods 1 and 2 while Figure 6 shows the same value for Method 3 (block of two strips). By comparing these three figures, it can be concluded that the best results were achieved using Method 2 in which the tower points were included in the combined strip adjustment. It is interesting to note that the RMSE values of the Z coordinates of check points obtained from Method 2 is similar to those of the X or Y coordinates.

Tables 7 and 8 show the reliability measures for image coordinates and GPS coordinates of exposure stations for the various methods obtained from the real data. Including the tower points in a single strip adjustment increases the reliability of the strip (Tables 7 and 8).

Conclusions

GPS-controlled strip triangulation was carried out using geometric constraints of man-made structures (power towers) to replace the ground control points needed to recover the roll angle of the camera. The results obtained from the simulated and real data show that if kinematic GPS can provide decimetre accuracy for the camera exposure stations, then the strip adjustment can be done without any ground control points as long as the datum transformation is known. Normally, two or three strips of photography are taken to recover the roll angle of the aircraft and to improve the geometry of a single strip. This new technique for single strip adjustment eliminates the need for multiple strips of photography and reduces both the time and the cost of the mapping project.

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