Classifying Hills and Valleys in Digitized Terrain

Terrence Cronin

Abstract
A technique is presented to differentiate hills from valleys, using a contour map. Unlike previous classification techniques based on a gridded representation of terrain, the new method is based on the containment relationship of nested contours. The primary contribution of this paper is an innovation to accommodate contours that leave the edge of the map. Because these contours are ambiguous with respect to interior and exterior, they warrant special treatment during containment testing operations. Their proper interpretation results in a more comprehensive classification product. Hills are constructed from the top down, by starting with a summit contour and incrementally annexing contours at lower elevations that contain the summit. Conversely, valleys are built from the bottom up, by annexing contours at high elevations that contain the basin contour. Construction of hills and valleys terminates when containing contours cease to exist.

Introduction and Statement of the Problem
Many mapping applications require subdivision of terrain into features that are intuitively obvious to a human observer or accessible to an automated spatial reasoning process. The U.S. Army defines primary terrain features to be hills, valleys, depressions, saddles, and ridges, and secondary features to be spurs, draws, cliffs, cuts and fills (U.S. Army, 1993). The motivation of this paper is to describe an automated method to extract three of those primary features — hills, valleys, and depressions — from a contour map. Hills, valleys, and depressions may be perceived respectively as the inverted bowls, partial bowls, and bowls of a landform.

For the purposes of this paper, the word "hill" refers to a landform on the surface of the Earth that is more convex (shaped like an inverted bowl) than it is concave (shaped like a bowl). Conversely, a valley is more concave than convex. This is not to say that a hill cannot contain a valley: consider a hanging canyon, carved by glaciers, on the side of a mountain. Likewise, a valley may contain a hill: consider the many islands of the St. Lawrence River, or a cinder cone within the caldera of a volcano.

Unfortunately, terrain classification is a subjective process at best. The geographic terms hill, valley, saddle, depression, spur, draw, ridge, and cliff are linguistic entities, perceived and interpreted by the map observer. His perception is limited to the scale and clipping region of the map environment. There is no established, standardized methodology to evaluate the performance of an automated terrain classification algorithm. Other than comparing algorithmic output to a consensus of human experts, there is no way to determine the quality of a terrain classification product. This shortcoming notwithstanding, attempts have been made by computer scientists, engineers, and geographers to automate the process of determining the boundaries of terrain features in digitized terrain databases.

This paper is organized into six sections. The first section serves as an introduction and statement of the problem. The second section discusses previous work on terrain classification, particularly as it relates to hill-valley differentiation. The third section describes the evolution of previous work dealing with graph theoretic representation of a contour map. The fourth section describes innovations designed to assist in classifying terrain with a contour map. The fifth section provides a set of examples that illustrate the results of the new algorithm when applied to a variety of contour maps. The sixth section presents conclusions and suggests directions for future research.

Previous Work on Terrain Classification
Most previous terrain classification research has dealt with the problem of deriving drainage networks from raster elevation files. Drainage network extraction is a mature technology with a well-developed literature. See, for example, Jenson and Domingue (1988), Skidmore (1990), Tribe (1991), Meisels *et al.* (1995), Schmid-McGibbon (1995), and Bennett and Armstrong (1996). The dual of a drainage network is a ridge line. Drainage and ridge line networks are lineal features, the former representing bottoms of valleys, and the latter crests of hills.

Although geographers and computer scientists have performed limited terrain classification work for decades, no previous automated method has been successful at identifying the terrain features listed in U.S. Army FM 21-26 (U.S. Army, 1993). Nor, for that matter, has any method completely solved the simpler problem of hill and valley classification. The topic of hill and valley classification is less developed than that of drainage network extraction, because it is a non-trivial task to locate the boundary between an elevated landform and a basin. Because hills and valleys are areal features, they encompass much larger regions than drainage networks. There has been limited success in extracting hills and valleys from the three most commonly implemented types of terrain databases: gridded raster, triangulated irregular network (TIN), and contour map.

Falcidieno and Spagnuolo (1991) visually distinguished hills from basins, using a triangulated irregular network (TIN). In a TIN, critical points of an elevation height field are used as vertices of triangles. Each triangle is classified as being concave upward or convex downward, based on the spatial relationship of edges between neighboring triangular facets. To visualize the surface, convex triangles are color-coded with a light shade and convex triangles with a dark shade. The method is subject to several visual artifacts, including representation of a continuous surface with a faceted network of triangles, and an aliased boundary between hill and basin areal features caused by the juxtaposition of alternating triangular vertices.

Graff and Usery (1993) developed an algorithm to subdi-

CECOM RDEC Intelligence and Information Warfare Directorate, ATTN: AMSEL-RD-IW-TP, Ft. Monmouth, NJ 07703-5211 (cronint@mail1.monmouth.army.mil).

Photogrammetric Engineering & Remote Sensing
Vol. 66, No. 9, September 2000, pp. 1129–1137.

0099-1112/00/6609–1129$3.00/0

vide a digital elevation model (DEM) representation of terrain into regions of mounts (elevated areas) and non-mounts. A DEM is a gridded raster representation of terrain. This work is noteworthy because it was the first to simplify the terrain classification problem by extracting general features first, and deferring decision on more refined features until later. The algorithm was based on locating maximum points of elevation in the DEM file and computing slope for adjacent DEM points. Prior to running the algorithm, a panel of humans was polled to establish ground truth for mount versus non-mount areas. Compared to the human consensus, the classification algorithm performed well in moderate to high relief areas with well-defined elevated features, but was less successful in low relief areas, or with maps containing elongated narrow ridges or broad, flat-topped mounts.

Kweon and Kanade (1994) extracted hills and valleys from a contour map, although they referred to hills as "peaks" and valleys as "pits." Their method relied on a data structure called a topographic change (TC) tree, which is an extension of the contour tree introduced by Roubal and Poiker (1985). The TC technique was applied to a contour map derived from a DEM, to label hills and valleys. Although the method performed well on databases consisting of closed contours and certain open contours confined to specific regions of a map, the issue of ambiguous open contours was not addressed. An ambiguous open contour is one that is not completely contained by the map, because it leaves the map edge at two distinct locations, frequently on opposite sides of the map. Consequently, the landform classification process was at times incomplete, resulting in limited labeling of hills and valleys covering only a portion of the entire map.

Blaszczynski (1997) developed a differential gradient method to subdivide a DEM surface into concave (bowl-shaped), convex (inverted bowl), and flat regions. By employing a window of variable size, the method visually portrayed the boundary lines between concave and convex regions of the DEM. An interesting result was that windows of smaller size generated a product with larger convex regions; conversely, windows of larger size generated larger concave regions. In geographic terms, this means that, as the size of the window increases, the spatial extent of hills shrinks, with a corresponding expansion in the extent of valleys. Selection of an appropriate window size remains an open issue.

Previous Work on Graph Theoretic Representation of a Contour Map

There has been considerable previous work to represent a contour map as a graph-theoretic data structure. A graph is comprised of nodes (represented as circles) connected by arcs (represented by lines). Generally, the arcs represent a binary relationship defined on the nodes. Various strategies have been proposed to transform a contour map into a graph. For a comprehensive survey, refer to Sircar (1991). The strategies differ in the way in which contour map objects and their relationships are represented as nodes or arcs of the graph.

Boyell and Rushton (1963) were the first to publish the mapping of contour lines into arcs and inter-contour regions into nodes. Morse (1968; 1969) mapped contour lines into nodes and inter-contour regions (called junctors) into arcs; thus, a contour line is spatially related to another by the region between them. Merrill (1973) mapped inter-contour regions into nodes and contour lines into arcs, and stressed the importance of topological containment. Roubal and Poiker (1985) mapped contour lines into nodes and adjacency of contours into arcs, but ignored inter-contour regions. Mark (1986) mapped contours and inter-contour regions into separate nodes, and used arcs to represent the adjacency relationship between contours and regions. Sircar (1991) enhanced Mark's method by exploiting elevation to impose partial ordering on adjacent contours. Kweon and Kanade (1994) extended the Roubal-Poiker concept by creating a topographic change tree to detect boundaries of hills and valleys. Cronin (1995) mapped both a contour line and its enclosed regions into a node, and the containment property of contours, ordered by elevation, into arcs. A summary of previous work dealing with the graph theoretic representation of contour maps is presented at Table 1.

An Improved Algorithm to Extract Hills and Valleys from a Contour Map

This section extends the landform classification method introduced by Cronin (1995), by elaborating upon methods introduced in the original work. First, an extension is described to address contours that leave the map's edge. Then, the mechanics of hill and valley classification are discussed, after incorporating the extension into the original method. With this technique, contours and their enclosed regions are represented as nodes, and the containment relationship between adjacent contours is represented as arcs of a data structure called the contour containment graph.

The Ambiguity of Open Contours

One of the main barriers to successful implementation of contour-based terrain classification has been the ambiguity of open contours. A topographic contour may be closed or open. A closed contour is completely contained by the map, and has identical first and last coordinates. In contrast, an open contour exits the map at two distinct locations along the map's edge, and has different first and last coordinates. For a closed contour, the concepts of inside, outside, and containment are well understood. The same is not true of an open contour, because an observer cannot know the connectedness of a contour that travels beyond the edge of the map. For example, consider a contour that intersects the left edge of a map in two places. Does the contour close upon itself just beyond the map's edge, or does it double back around the rectangular border of the map and connect in two places to another contour of the same elevation at the map's right edge? These questions cannot be answered simply by studying the informational content located within the border of the map. Therefore, an open contour, due to its ambiguous nature, warrants special processing treatment with regard to the concept of containment.

Note that an open contour, by intersecting the map's edge at two locations, divides the map into exactly two regions. The regions are complements of each other, in the sense that their union is the entire map (Figure 1). To implement the concept of containment, a decision must be made regarding which region represents the contour's interior and which the exterior. Because this decision is crucial for automated construction of hills and valleys, it is important that the logic closely mirrors the analytical skills used by humans when locating landforms on a contour map.

To resolve the open contour dilemma, this paper advocates using the region with smaller area as the contour's interior (Figure 1). For each of the two regions formed by an open contour with the map's edge, the area may be computed and compared to the area of the other region, to determine which is smaller. This is possible because in the digital domain a contour is represented as a set of discrete coordinates. A formula to compute the area bounded by a contour having coordinates $\{(x_0, y_0), (x_1, y_1), \ldots, (x_{n-1}, y_{n-1})\}$ is as follows:

$$A = \frac{\sum_{i=0}^{n-1} (x_i y_{i+1} - y_i x_{i+1})}{2}. \quad (1)$$

Table 1 Evolution of Graph-Theoretic Representation of Contour Maps.

Author (date of publication)	Nodes of Graph	Arcs of Graph
Boyell and Rushton (1963)	Inter-contour regions	Contour lines
Morse (1968; 1969)	Contour lines	Inter-contour regions
Merrill (1973)	Inter-contour regions	Contour lines ordered by containment
Roubal and Poiker (1985)	Contour lines	Adjacency of contours
Mark (1986)	Contour lines and inter-contour regions	Adjacency of contours
Sircar (1991)	Contour lines and inter-contour regions	Adjacency and partial ordering on elevation
Kweon and Kanáde (1994)	Contour lines and inter-contour regions	Topographic change
Cronin (1995)	Contour lines and their enclosed regions	Contour containment based on elevation and areal constraints

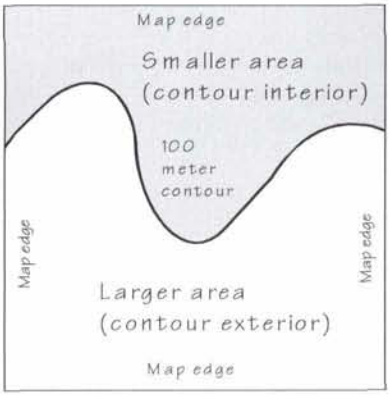


Figure 1. Interior of an open contour as the region of lesser area.

The motivation for choosing the smaller region as the interior of an open contour is based on an observation about the relative size of terrain features on a contour map. In general, contour maps contain multiple hills and valleys, with each hill or valley occupying less than half the area of the map. This observation is based solely on the experience of the author, who has interpreted a variety of topographic maps. Each hill or valley is usually constrained to a relatively small portion of the map, specifically to less than half the map. There are, of course, exceptions. For example, if one zooms in to fill the map's rectangular clipping region with a single hill or valley, correct interpretation is confounded. However, because the act of zooming increases ambiguity even for human interpretation, an assumption is made that multiple features of manageable scale are present on a contour map.

The main issue arising from the minimal area heuristic is that terrain classification is dependent upon map scale. For example, consider a map of the Himalayas depicting Mt. Everest as a feature of relatively small area when compared to the area of the entire Himalayan range shown by the map. In this case, most or all of the contours representing Mt. Everest would be closed, so that the entire nested contour complex would be considered a hill structure. Now, consider a zoomed-in map showing only the north face of Mt. Everest, with no other mountains represented within the map's rectangle. Due to the minimal area heuristic, the upper half of the north face will be considered a hill, whereas the lower half will be considered valley. Thus, hill-valley classification is dependent upon map scale. Note, however, that closed contours are labeled consistently no matter what the map scale. Open contours are the source of uncertainty.

Constructing the Contour Containment Graph

Every closed contour on a map encloses a unique region. Based on the minimal area heuristic described above, an open contour also encloses a unique region (the smaller of the two areas it forms with the map's edge). Because containment is thereby defined not only for closed contours, but also for open contours, it is feasible to proceed with hill and valley construction based on containment testing. Each node of the contour containment graph represents a region consisting of the union of a contour and its interior. A vertical link pointing downward in the graph represents the concept "the region above is contained by the region below," and a vertical link pointing upward represents the concept "the region below contains the region above."

Prior to building the contour containment graph and classifying terrain as hills or valleys, the contour map is represented as a list of contours. In this paper, a contour is represented by a lower case c. Each contour in the list consists of an index number, an elevation, and a set of x-y pairs (coordinates): i.e.,

$$c = \{\text{index\_number}, \text{elevation}, \{(x_i, y_i), i = 1, 2, \ldots, m\}\}. \quad (2)$$

The contour list is sorted from highest to lowest elevation. Equation 3 formalizes the list structure, where el(c) represents the elevation of contour c. Contours corresponding to the tops of hills are located towards the front of the list, and those corresponding to the bottoms of valleys are located towards the back of the list. With sorted contour list in hand, hill construction proceeds from the top down, and valley construction from the bottom up (Figure 2).

$$contour\_list = \{c_i, i = 1, \ldots, n; \textit{ such that } el(c_i) \geq el(c_{i+1})\} \quad (3)$$

Contours that comprise hill structures are subject to the following constraints:

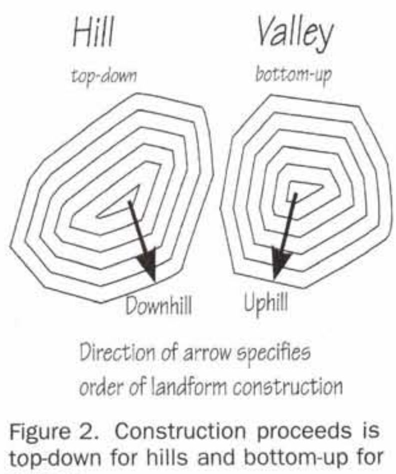


Figure 2. Construction proceeds is top-down for hills and bottom-up for valleys.

$h1.\ c_{i_1} \subseteq c_{i_2} \subseteq \cdots \subseteq c_{i_k}$

$h2.\ el(c_{i_1}) > el(c_{i_2}) > \cdots > el(c_{i_k})$

$h3.\ i_1 < i_2 < \cdots < i_k$

$h4.\ Area(open\_contour) \leq (l^*w)/2$

The first constraint states that the containment hierarchy of a hill proceeds from summit to base (the summit contour is contained by a contour located one contour interval downhill, etc.). The second constraint states that the contours comprising a hill are discovered from the top of the hill down, from highest elevation to lowest elevation. The third constraint states that the contours of a hill are found from left to right in the contour list (recall that the contour list is ordered from highest to lowest elevation). The fourth constraint specifies that the area of the region formed by an open contour that is part of a hill does not exceed one half the area of the map's rectangle.

Contours that comprise valley structures are subject to the following constraints:

$v1.\ c_{i_1} \supseteq c_{i_2} \supseteq \cdots \supseteq c_{i_k}$

$v2.\ el(c_{i_1}) < el(c_{i_2}) < \cdots < el(c_{i_k})$

$v3.\ i_1 > i_2 > \cdots > i_k$

$v4.\ Area(open\_contour) \leq (l^*w)/2$

The first constraint states that the containment hierarchy of a valley proceeds from basin to head wall (the basin contour is contained by a contour located one contour interval uphill, etc.). The second constraint states that the contours comprising a valley are discovered from the bottom of the valley up, from lowest elevation to highest elevation. The third constraint states that the contours of a valley are found from right to left in the contour list (recall that the contour list is ordered from highest to lowest elevation). The fourth constraint specifies that the area of the region formed by an open contour that is part of a valley does not exceed one half the area of the map's rectangle.

The two sets of constraints partially specify the design of an algorithm to construct hills and valleys. By processing the contour list from left to right, hills are constructed. Then, by processing the list from right to left, valleys are discovered. With the addition of logic to annex subordinate hills to primary hills, subordinate valleys to primary valleys, subordinate hills to primary valleys, and subordinate valleys to primary hills, a complete classifier emerges. Elaboration follows.

Constructing Hills

Hills are built from the top down, by nesting incrementally lower elevation contours about a summit contour, until no lower contours containing the summit exist, or the rectangular boundary of the map causes construction to be terminated. The first hill is constructed by beginning with the contour of highest elevation, assigning it a label, and searching the contour list for a contour at the next lower elevation that contains the first contour. If such a contour is located, it is called the *current containing contour.* The current containing contour is assigned the same label as the contour it contains, and is subsequently renamed the *previous containing contour.* Now, the search resumes by looking for a contour at a lower elevation that contains the previous containing contour. Eventually, the hill structure is completed, because no contour other than the rectangular boundary of the map contains the previous containing contour.

Additional hills are constructed with similar logic, with one notable exception. When completed, each new hill is checked for containment within previously constructed hills. In this way, secondary features such as pillars, spires, and mounds that are subordinate to a primary hill are associated with the primary hill, and are labeled as such. Thus, a secondary feature has its own identifying label, and another label to associate it with a primary feature. If a subsequent hill is not contained within a previously constructed hill, then it is independent, standing apart from other hills constructed thus far.

Constructing Valleys

Valleys are built from the bottom up, by nesting incrementally higher elevation contours about a basin contour, until no higher contours containing the basin exist, or the rectangular boundary of the map causes construction to be terminated. The logic used to build the first and subsequent valleys is similar to the logic used to construct hills, except processing proceeds from lowest elevation to highest elevation. When constructing valleys, it should be noted that basin contours are open contours that intersect the edge of the map, usually the same edge, but sometimes multiple edges.

Constructing Depressions

Depressions are constructed similarly to valleys. However, the basin contour for a depression is closed and completely contained by the map. Depressions are rare phenomena when compared to hills and valleys, and are not normally present on a contour map. Despite their anomalous nature, depressions are considered to be one of the primary terrain features, as listed in U.S. Army Field Manual 21–26 (U.S. Army, 1993).

Locating Hills Within Valleys, and Valleys Within Hills

When hill and valley construction terminates, yet another level of processing remains. To capture hills that are contained by valleys, the set of constructed valleys is searched to ascertain whether hills are contained within their regions. Similarly, the set of constructed hills is searched to determine whether valleys are contained within them. This stage of processing guarantees, for example, that a ravine is associated with a specific hill structure, or that a mesa is associated with the valley that contains it.

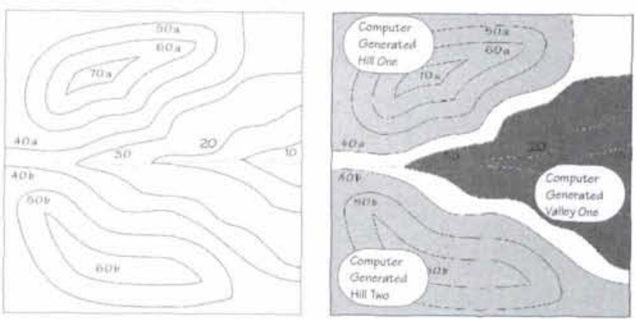
The Ambiguity of the Transition Region between Features

Note that labeling of hills and valleys does not constitute a complete space-filling classification of the map. That is, after hill and valley construction terminates, an unlabeled space remains, constituting a gap between adjacent contours, which is neither hill nor valley, but rather the transition region between them. If a space-filling algorithm is required, the transition region may be divided into multiple subregions, based on proximity (Voronoi diagram) to the nearest hill or valley. As a side issue, the transition region is a likely location for saddles or mountain passes.

A crisp line of demarcation between two hills, two valleys, or a hill and a valley is difficult to obtain. This is because it is not known where an inflection point lies between adjacent contours, such as two bottom contours of neighboring hills. One possible remedy to the ambiguity is to go back to the gridded data from which the contours were extracted, in order to find break points between terrain that is convex down versus concave up.

Examples of Hill and Valley Construction

This section contains examples of the new algorithm applied to four contour maps. The first example is a simple simulated terrain that illustrates the capability of the new algorithm to handle open contours that leave the edge of the map. The second example demonstrates the behavior of the algorithm in an area of high relief: Grand Teton National Park, Wyoming. The third



(a) (b)

Figure 3. (a) Contour map of simulated terrain. (b) Classification of hills and valleys for simulated terrain.

example provides an example of a hill contained within a valley, that is, the cinder cone within the caldera of Mount St. Helens, Washington. The fourth example illustrates the utility of the minimal area heuristic when partitioning a map into regions of hills and valleys, for Killeen, Texas.

Example 1

Consider the simulated terrain of Figure 3a. The map is hand crafted, with a 10-meter contour interval. There are ten contours on the map, indexed by labels 10, 20, 30, 40a, 40b, 50a, 50b, 60a, 60b, and 70a. The numeric magnitude of the label represents elevation in meters. In a preprocessing step, the contours are sorted into a list, from highest to lowest elevation, as shown at Table 2.

The process of hill construction begins with selection of contour 70a, at the front of the contour list. The new hill is named "Hill One," because hill construction has just been initiated. Table 2 is searched to the right for a contour that contains contour 70a. The contour to the immediate right, contour 60a satisfies the test, so it is annexed to the hill structure. Search continues to the right to seek a contour that contains contour 60a. Because contour 50a passes, it is incorporated into the hill structure. At this time, Hill One is comprised of the nested contour complex 70a-60a-50a. Note that all three contours are closed and do not intersect the map's edge.

Now the table is searched to find a contour containing contour 50a. Note that contour 40a (an open contour) forms two regions with the edge of the map. As described in the previous section, the smaller of the two regions is used to represent the interior of contour 40a. The smaller region does indeed contain contour 50a, so it is annexed to Hill One. Now the algorithm searches for a contour at elevation 30 meters that contains contour 40a. Although there is a 30-meter contour in the database, its interior (the smaller of the two regions formed by contour 30 with the map's edge) does not contain contour 40a. Hence, construction of Hill One terminates, after determining that it consists of contours 70a, 60a, 50a, and 40a, from summit to base.

Similarly, Hill Two is constructed, after determining it consists of contours 60b, 50b, and 40b. After Hill Two is constructed, no other hills are found. Hill Two is subsequently tested for subordination to Hill One, but fails the test. When hill construction terminates, a decision is made that the map contains two different hills, each spatially independent of the other.

Next, valley construction is initiated from the bottom up, beginning with the lowest elevation contour. The 10-meter elevation contour is selected as a tentative basin contour, and the algorithm searches for a 20-meter elevation contour that contains contour 10. Contour 20 passes the test. Similarly, contour 30 is found to contain contour 20. Now, the algorithm searches for a 40-meter elevation contour that contains contour 30. Neither the minimal area region formed by contour 40a nor the minimal region formed by contour 40b contains contour 30. Thus, construction of Valley One is terminated after determining that it consists from bottom to top of contours 10, 20, and 30. Because no other valleys exist on the map, valley construction terminates.

Finally, a cross check is performed to determine whether Hill One or Two is contained within Valley One. This test is performed by checking to see if the lowest elevation contour of either hill is contained within some contour of Valley One. In both cases, the test fails. Then a similar test is performed to determine whether Valley One is contained within Hill One or Hill Two, but again the test fails. Thus, for this simple example, there are no hills subordinate to valleys, nor are there any valleys subordinate to hills. The hill-valley classification for the contour map of Figure 3a is shown at Figure 3b.

Example 2

Refer to Figure 4a, a contour map of the Middle and Grand Teton area in Wyoming. The source is USGS, the scale is 1:24,000, the contour interval is 80 feet, and the index contour interval is 400 feet. Figure 4b is a close-up of the Middle Teton, in which the contours of the figure have been assigned index numbers. Assume that hill construction is underway, and that the hill corresponding to the Grand Teton has already been constructed, similar to Hill One of Example 1 above. Beginning at the top of the Middle Teton, contour number 1 is selected, and is assigned the label "Hill Two" (because construction of the higher elevation Grand Teton has already been completed, and has used the label "Hill One"). The contour list is searched for a contour at a lower elevation that contains contour 1. Contour 2 satisfies the containment criterion, so it becomes the current containing contour. Now, contour 2 is assigned to be the previous containing contour, and the contour list is searched for a contour that contains contour 2. Contour 3a satisfies the criteria, and is annexed to the Hill Two structure. Eventually, Hill Two is completed after annexation of contour 15, which is not contained by any other contour, other than the edges of the map. Note that the label "Hill Two" may be replaced at any time by the label "Middle Teton," which is the actual name of the feature. The replacement might be performed by a human user, or by an automated process with access to a geo-spatial gazetteer.

Equations 4 through 9 represent the decomposition of Hill Two into branches of the contour containment graph (explicit representation of the h1 constraint discussed in the previous section). The binary relationship expressed by the equations may be interpreted as both contour containment and set membership. For example, the region contained by contour 1 is a subset of the region contained by contour 2. The primary elevated landform of Hill Two is represented at Equation 4. The other equations represent containment hierarchies of smaller hill features subordinate to the Middle Teton.

$$1 \subseteq 2 \subseteq 3a \subseteq 4 \subseteq 5 \subseteq 6a \subseteq 7a \subseteq 8 \subseteq 9a \subseteq 10 \subseteq 11 \subseteq 12 \subseteq 13a \subseteq 14 \subseteq 15 \quad [4]$$

$$3b \subseteq 4 \subseteq 5 \subseteq 6a \subseteq 7a \subseteq 8 \subseteq 9a \subseteq 10 \subseteq 11 \subseteq 12 \subseteq 13a \subseteq 14 \subseteq 15 \quad [5]$$

Table 2. Contour List for Simulated Terrain of Figure 4a

Position in list	1	2	3	4	5	6	7	8	9	10
Label of contour	70a	60a	60b	50a	50b	40a	40b	30	20	10

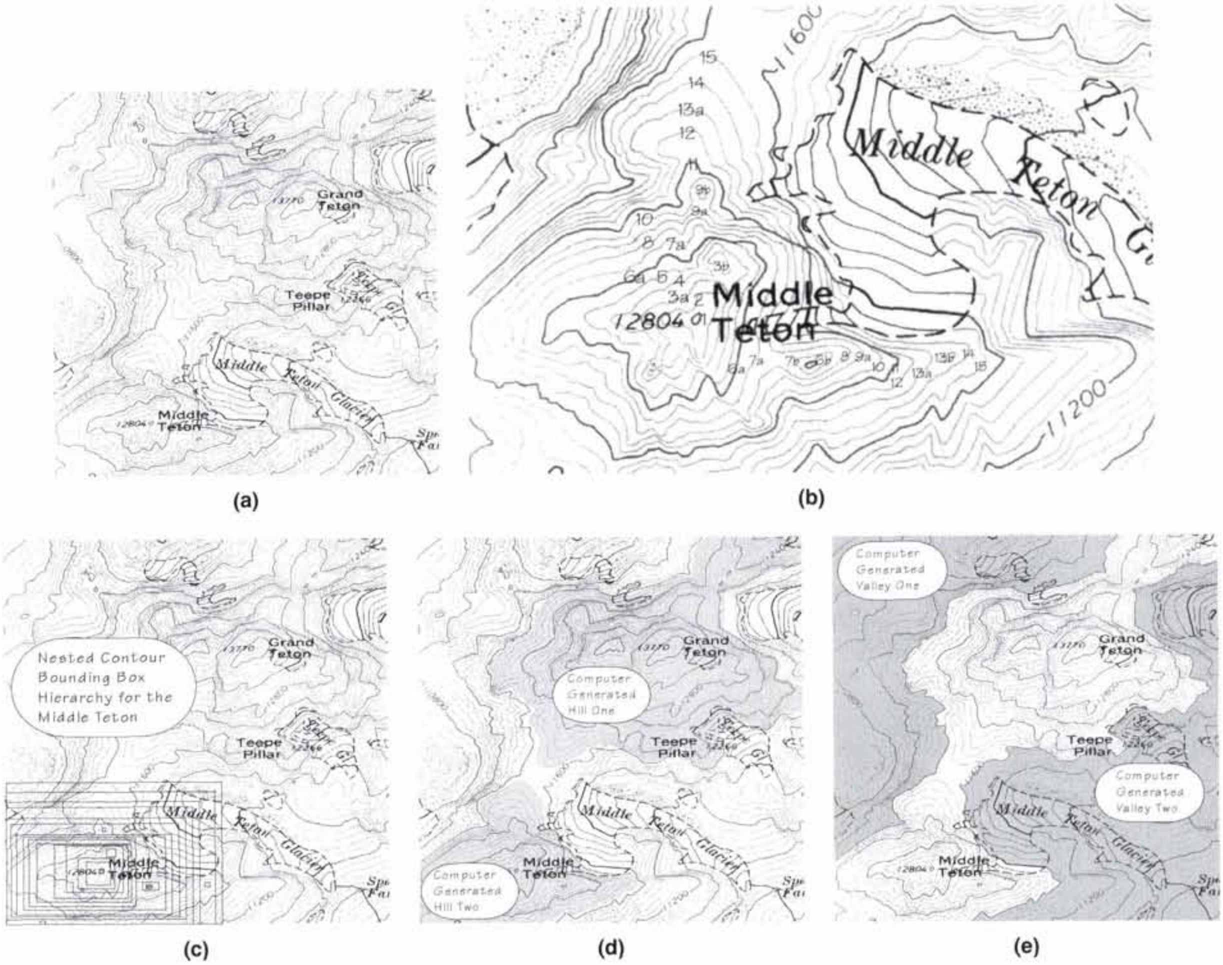


Figure 4. (a) Contour map for Grand and Middle Teton. (b) Contour indexing for Middle Teton. (c) Minimum bounding rectangle hierarchy for Middle Teton. (d) Labeled hills for Teton map. (e) Labeled valleys for Teton map.

$$3c \subseteq 4 \subseteq 5 \subseteq 6a \subseteq 7a \subseteq 8 \subseteq 9a \subseteq 10 \subseteq 11 \subseteq 12 \subseteq 13a \subseteq 14 \subseteq 15 \quad [6]$$

$$6b \subseteq 7b \subseteq 8 \subseteq 9a \subseteq 10 \subseteq 11 \subseteq 12 \subseteq 13a \subseteq 14 \subseteq 15 \quad [7]$$

$$9b \subseteq 10 \subseteq 11 \subseteq 12 \subseteq 13a \subseteq 14 \subseteq 15 \quad [8]$$

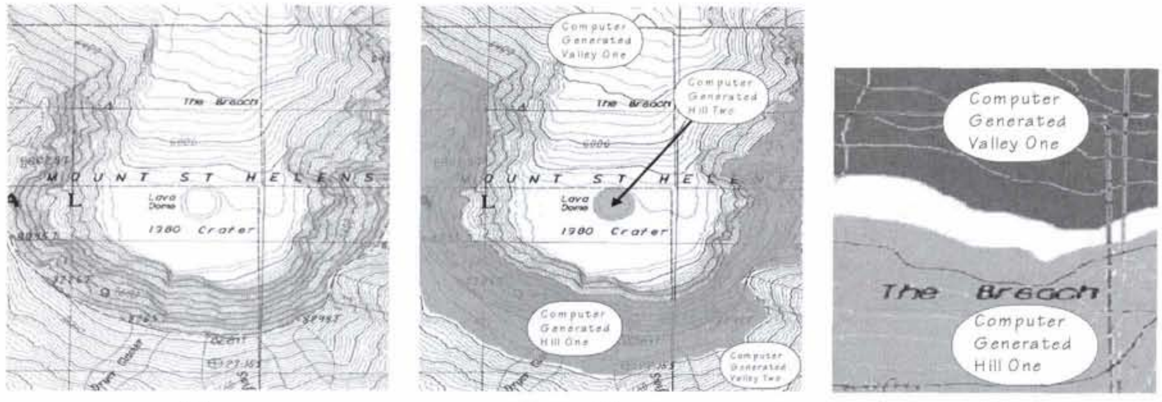
$$13b \subseteq 14 \subseteq 15 \quad [9]$$

To improve performance of containment testing, it is frequently possible to exploit the minimum bounding rectangle (MBR) of each contour, rather than appealing to the full power of a general point-in-polygon algorithm. The MBR hierarchy for the Middle Teton is illustrated at Figure 4c. Note that simple containment testing of one rectangle within another, from summit to base, yields a correct contour hierarchy for the Middle Teton. However, there are cases when the MBR technique fails, perhaps the simplest to visualize being a contour enveloped by a "horseshoe-shaped" contour. Clearly, in this instance, the enveloped contour is not contained within the enveloping contour. Future research will investigate under what conditions the MBR technique may be leveraged successfully.

Hill classification for the Teton map is illustrated at Figure 4d, and valley classification at Figure 4e. Note the many open contours on the map, some associated with the two hill structures and some with the two valleys. For the open contours, the policy of using the smaller area region with respect to the map's edge has resulted in an intuitively satisfying classification. The labeled features have names in the real world. Hill One is known as the Grand Teton-Mount Owen landform (Mount Owen is the subordinate hill feature at the upper right corner of the map). Hill Two is the Middle Teton, Valley One is called Cascade Canyon, and Valley Two is called Garnet Canyon.

Example 3

Refer to the contour map of Figure 5a, corresponding to Mount St. Helens, Washington. The source is USGS, the scale is 1:24,000, the contour interval is 40 feet, and the index contour interval is 200 feet. In addition to its depiction of extreme relief, this database was chosen to demonstrate a hill subordinate to a valley. Note the breach at the top central (north) portion of the mountain. This area was the region of major lava flow during the 1980 eruption of Mount St. Helens. The breach is classified as a valley by the contour containment algorithm, because it consists of a basin contour at the top right center of the map, enveloped by a series of contours at increasing elevation. The valley's head wall is about halfway up the south face of the crater. It stops about halfway because the minimal area heuristic assigns the upper part of the crater to a hill. The



(a) (b) (c)

Figure 5. (a) Contour map for Mount St. Helens. (b) Hills (gray) and valleys (white) for Mount St. Helens map. (c) High ground (light gray) and low ground (dark gray) at breach area.

breach contains the cinder cone (lava dome) of the volcano. However, the cinder cone is classified as a hill by the algorithm, since it is comprised of two nested contours, the inner contour being at a higher elevation than the outer contour. The hill-valley classification for Mount St. Helens is shown at Figure 5b. Elevated landforms are shaded in gray, whereas valleys remain unshaded.

A close-up of the breach illustrates the scale-dependence of the terrain classification algorithm. Observe that Figure 5c contains no closed contours, indicating that every contour is a source of ambiguity. The classifier creates a hill (high ground with respect to the map's rectangle) in the lower portion of the map, a valley (low ground) at the top of the map, with a transition region between. The classification is different from that developed for Figure 5b, which is a smaller scale map.

Example 4

Refer to the contour map of Figure 6a, corresponding to Killeen, Texas. The source of the terrain is the National Imagery and Mapping Agency (NIMA), the contour map was generated with the Generic Mapping Tools (GMT) (Wessel and Smith, 1991), the map scale is 1:50,000, and the contour interval is 10 meters. The database is Digital Terrain Elevation Data (DTED), Level 2. The actual height field for this region is a two-dimensional 901 by 901 array of elevation values at approximately a 30-meter post spacing, representing a one-quarter degree cell of the Earth's surface. GMT outputs contour data to a file, with each contour assigned an elevation value in meters and a list of coordinates (*x*-*y* pairs). Contours within the output file are sorted on increasing elevation to create the contour list.

A gray-scale illustration of the Killeen height field is shown at Figure 6b. The lower elevation portions of the region are located at the right side of the map. The light gray regions are a series of reservoirs. There are three primary elevated landmasses: the central highlands depicted in light and medium gray, the northern highlands above the large diagonally oriented reservoir, and a small highlands just north of the southernmost reservoir. Both the central and the southern highlands are classified as subordinate features to the Hill One landmass of Figure 6c. The northern highlands are subordinate to the Hill Two complex at the top of the map. Note that Hill One and Hill Two are segregated by a valley, a phenomenon similar to the simulated terrain of Example 1 above. There are two additional hill structures found, along the bottom edge of the map neighboring the reservoir, but for clarity of illustration, they are not labeled. The white areas of Figure 6c are classified as valleys by the algorithm, with no distinction made among the reservoirs, because a continuous contour envelops them all.

For the Killeen database, some additional work has been performed to locate two of the secondary features in U.S. Army FM 21-26 (U.S. Army, 1993). The two secondary features are spurs and draws. Spurs are to draws as hills are to valleys, because spurs are elevated landforms and draws are at lower elevations. Usually, but not always, spurs and draws are located at approximately right angles to the ridge line of a hill. Also, spurs and draws are often complementary, with draws extending from a valley up the side of a hill. Located adjacent to draws, in an interleaved fashion, are spurs that protrude from the side of a hill.

To classify spurs and draws, a concavity code was developed to characterize the shape of individual contours (Cronin, 1999). The code was inspired by a seminal work (Attneave, 1954) suggesting that most of the information in a curve is located at areas of high curvature. For spur-draw classification, this means that if a contour is curved outward (convex), a spur may be present in the terrain. Conversely, if a contour is curved inward (concave), a draw may be present. A syntactic parsing algorithm that exploits the contour concavity code was used to extract spurs and draws. A graphic illustrating the results of spur-draw classification is shown in Figure 6d. Spurs are color-coded white and draws black. Spurs and draws are particularly visible along the edges of the reservoirs, at cliffs that accentuate contrast.

Conclusions and Suggestions for Future Research

A new algorithm has been presented to classify hills and valleys in digitized terrain. The algorithm is based on the containment relationships of contours comprising hills and valleys. The primary contribution of this paper is a technique to accommodate open contours that leave the edge of the map. Unlike closed contours with well-defined interiors, open contours are problematic because it is difficult to determine which region of the map represents their interior. This paper defines the interior of an open contour to be the smaller of the two regions the contour forms with the map's edge. The ability to test open contours for containment within other contours is beneficial, because it results in a more comprehensive classification product than has previously been obtainable from a contour map.

The classification technique as currently implemented is scale-dependent. It is also somewhat arbitrary in the sense that

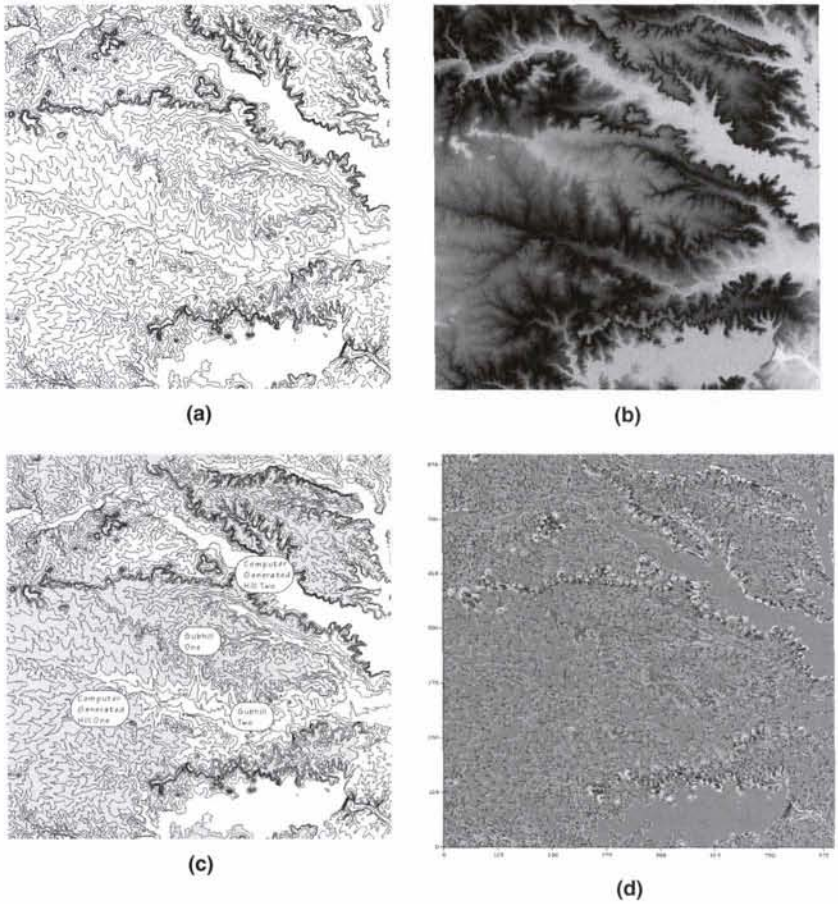


Figure 6. (a) Contour map for Killeen, Texas. (b) Grayscale map of height field for Killeen map. (c) Hills (gray) and valleys (white) for Killeen map. (d) Spurs (white) and draws (black) classified for Killeen map.

it resolves the interior of an open contour to be the region of smaller area with respect to the map's rectangle. These two issues require further investigation. Also, there is an urgent need to arrive at an evaluation methodology for terrain classification algorithms, because determination of the boundary between hills and valleys appears to be subject to individual interpretation.

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(Received 16 April 1999; accepted 15 September 1999; revised 19 October 1999)